# 25. Thermodynamics and Economics

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**Abstract**

Thermodynamics and economics have developed independently through the last centuries. Only in the last three decades scientists have realized the close relationship between economics and physics. The name of the new field is econophysics:
In double-entry accounting the sum of monetary and productive accounts is zero.
In calculus monetary and productive accounts may be represented by Stokes integrals.
In engineering Stokes integrals lead to the two levels hot and cold of Carnot motors.
In production Stokes integrals lead to the two level process: buy cheap, sell expensive.
In economics the two level mechanism of capital and labour is called capitalism.

A heat pump can extract heat from a cold river and heat up a warm house.
A monetary circuit extracts capital from a poor population and makes a rich population richer!

A running motors gets hotter, the efficiency, the difference in temperatures, grows with time.
A running economy gets richer, the efficiency, the difference between rich and poor, grows.

**Keywords:** Ex ante, ex post, calculus based economics, econophysics, production, capitalism.

## 25.1 Introduction

Economics is a field of philosophy based on experience and tradition rather than on mathematical foundations. Economic laws are stated as principles that have been developed by different schools beginning with Adam Smith and classical economics in the late 18$^{th}$ century. *Economics* today is considered to be a field of *social science*, cooperation and competition as well as buying and selling is closely related to game theory and *psychology*. Business is entangled with contracts and public *law*, banking and money are tied to properties like hope and trust that are clearly part of *social sciences*.

However, if the early economists had stated their principles by calculus that had been developed by the British natural scientist Isaac Newton (1643 – 1727) and the German mathematician Gottfried Wilhelm Leibniz (1646 – 1716), economic science would perhaps be a completely different field of science. In the early days of natural production people worked in the fields to obtain food and shelter. Work, food, heat are measured in energy units, in kWh or in calories. Energy is an elements of natural science, of *biology*. Work, again, is an element of mechanics and indicates a close relationship to *physics.* Cooperation and competition are well-known from chemistry. People like atoms attract each other and cooperate, repel each other and dissociate, or are indifferent and integrate. Cooperation, segregation and integration have the same structure in social science and in chemistry. Only the forces are different: In chemistry we have electromagnetic forces between atoms, in societies we have social bonds like love, hate or indifference between people. Obviously, economics is linked to *chemistry.* Trade follows the law: buy cheap – sell expensive! Every shop has to buy a product at a low price and has to sell it at a higher price in order to make a profit. This process is repeated and repeated and requires labor to keep the economy running. If the price for buying and selling is the same, trade will stop. The same mechanism is known for heat engines or motors: Cold air is sucked in by the motor and hot air is blown out. This process is repeated and repeated and requires energy to keep the motor running. If the two temperatures of the motor are the same, the motor will stop. Production and trade apparently are closely linked to *engineering!* Banks, surprisingly, rarely use the term trust, they prefer the term risk, the reciprocal of trust: low trust is high risk and high trust means low risk. The term risk is again linked to statistics and to *statistical mechanics*. Accordingly, *economics* seems to be very close to the fields of *natural science*.

Indeed, since 1990 natural scientists have started to investigate economics, finance and social sciences by means of statistics, calculus and tools of theoretical physics. The new field has been called econophysics [1 - 9]. Presently, the name econophysics is mainly focused on finance. But many authors use the terms econophysics or complexity for the combination of social and natural sciences.

## 25.2 Neoclassical problems

There is a widely accepted economic theory, the neoclassical approach [10, 11]. However, this theory is not based on calculus and shows deficits that can be solved by calculus based theory:

### 25.2.1 Solow model

The Solow model of neoclassical theory claims income (Y) to be a function of capital and labour:

$$Y = F\ (K, L). \qquad (25.2.1).$$

Income (Y) is a function (F) of capital (K) and labour (L). The function F (K, L) is called production function.

Every investor would like to know how much he will earn in the investment period. But income (Y) is an *ex post* term, we can file our income tax only at the end of the year. Income can only be determined when the money is earned. In contrast a function (F) may be calculated at any time, in the beginning or at the end of the year. Functions (F) are called *ex ante* terms. *Ex ante* cannot be equal to ex *post*! Either we know beforehand or not. This means income (Y) cannot be a function (F):

$$Y \neq F\ (K, L)! \qquad (25.2.2).$$

This puzzle will be solved by exact *(ex ante*) and not exact *(ex post*) differential forms in calculus based economic theory.

### 25.2.2 The neoclassical misinterpretation of a monetary balance as a circular flow

Neoclassical theories consider monetary circuits as ring-like flows: Income (Y) flows from industry to households and flow back to industry as consumption costs (C). The surplus (S) flows from households to the bank, from where the money flows back into the economic circuit as investment (I) of industry.

However, this cannot be true, if we put in the numbers of the balance: A company pays 100 € per day to each worker and gets back only 90 € for consumption cost. The company has to withdraw $\Delta M$ = 10 € from the bank to be able to pay the workers 100 € again the next day!

This neoclassical model cannot work!

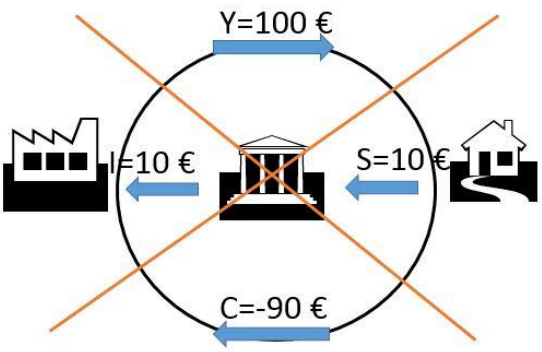


**Fig. 25.1** A monetary balance cannot be a closed circuit flow. If a company pays 100 € per day to each worker and gets back only 90 € for consumption cost, the company has to withdraw 10 € from the bank to be able to pay the workers 100 € again the next day! This neoclassical model cannot work! Again, this problem will be solved by Stokes integrals in calculus based economic theory.

## 25.3 The monetary balance

Balances are the fundament of economics. Every merchant has to look at his economic balance. A monetary balance is an account of numbers measured in monetary units like €, US $, £ etc. The account must indicate the economic unit, the person, the household, the company to which it refers. We may indicate the numbers by symbols and the economic unit by a suffix like $Y_H$ for income of a household, or $C_{In}$ for industrial costs, $\Delta M_X$ for the annual surplus of the company. The balance overlooks a well-defined time interval, like a day, a moth or a year.

E*xample 25.3*:

**A household (H) works in industry (In) earning $Y_H = 100$ € per day and spending $C_H = 90$ € for food and goods. The daily surplus is $\Delta M_H = 10$ €.**

We may summarize the balance of example 25.3.1 by an equation,

$$Y_H - C_H = \Delta M_H \qquad (25.3.1)$$

A positive balance ($\Delta M$) is a surplus; a negative balance ($\Delta M$) is a deficit.

### 25.3.1. The monetary balance as excel calculation

The excel sheet is the most popular tool for calculating a balance like in example (25.3.1):

| | | |
|---|---|---|
| **$Y_H$ =** | **100** | **€** |
| **$C_H$ =** | **-90** | **€** |
| **$\Delta M_H$ =** | **10** | **€** |

**Fig. 25. 2** Excel calculation. Examples for similar calculations are shopping receipts store or bank accounts.

### 3.2. The monetary balance as a spiral

The monetary balance may be repeated every day and is often considered as a cycle. But is this really a circle? If we put in numbers, we start at $Y_1 = 100$ € and get to costs $C_1 = 90$ €. This leaves a surplus $\Delta M_1 = 10$ € at the first day. At the next day we continue with $Y_2 = 100$ € and get to costs $C_2 = 90$ € with a surplus $\Delta M_2 = 10$ €. This leaves a surplus of $\Delta M_2 + \Delta M_1 = 20$ € at the second day. The surplus leads to a higher level each day, after n days the total surplus will be $\Delta M_n = n \cdot 10$ €. The continuous monetary balance is a spiral! With positive surplus the spiral will go up, with permanent deficit the spiral will go down, fig. 25.3.

**Fig. 25.3** A positive balance is a spiral that goes up, a negative balance is a spiral that goes down. $\Delta M$ is the lift after each round. The balance of example 3.1 may be regarded as a spiral going up by 10 € every day.

### 25.3.3 The monetary balance as a closed Stokes line integral

In chapter 3.1 we have discussed the monetary balance of households.


**Fig. 25.4.** The monetary circuit of households is a closed spiral, looking into the spiral .We may present the circuit in rectangular or circular shape.

We may interpret the monetary balance as a closed Stokes integral:

$$\oint \delta M = Y_H - C_H = \Delta M_H. \qquad (25.3.2).$$

If the monetary Stokes integral is positive, the spiral goes up after each round by $\Delta M_H$. If the monetary Stokes integral is negative, the spiral goes down after each round by $\Delta M_H$. Equation (25.3.2) is equivalent to eq. (25.3.1).

### 25.3.4 The double-entry balance

The double–entry balances considers the monetary and the productive and by adding both accounts: the sum is always zero. This equivalence of monetary and productive accounts is the origin of the word "balance" in accounting.

***a. The double-entry balance as an excel calculation***

The double-entry balance adds the monetary balance and the productive balance in monetary units by an excel sheet:

| | | | | | | |
|---|---|---|---|---|---|---|
| $Y_H$ = | 100 | € | | $W_H$ = | -100 | € |
| $C_H$ = | -90 | € | | $G_H$ = | 90 | € |
| $\Delta M_H$ = | 10 | € | | $\Delta P_H$ = | -10 | € |

**Monetary account + Productive account = 0**

In double-entry accounting, the monetary balance and the productive balance always add to zero. Households obtain income ($Y_H$) = 100 € by investing labour worth 100 €. They have costs ($C_H$) = – 90 € receiving goods worth 90 €. Accordingly, the sums always add up to zero!

***b. The double-entry balance in circuits***
Fig. 3.1 shows the sum of monetary (M) and productive (P) circuits of a household: the sum is zero. In this figure the household receives wages (+100 €) and gives away labor (- 100 €), the household pays consumption costs of (- 90 €) and obtain consumption goods for (90 €). Both circuits are non-zero and run into the opposite direction.

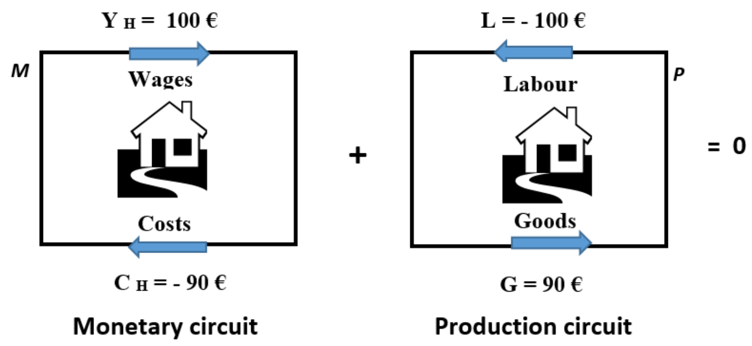


**Fig. 25.5** demonstrates the equivalence of monetary and productive circuits for a household. Both circuits run in opposite direction, the sum of both circuits is zero.

***c. The double-entry balance in spirals***
We may also interpret fig.3.3 as two spirals that wind in opposite directions. The monetary spiral winds upwards and the productive spiral winds downwards. The sum of both movements remains zero.
Courses on double entry bookkeeping also include accounts for deficits and credits. This makes bookkeeping more complex and difficult to handle, but the basic idea remains the same: all corresponding double entry accounts add to zero. This is the most important result of double entry bookkeeping, and we will now try to translate this important result into economic equations.

***d. The double-entry balance as fundamental integral of economics***
We may also express the results of double entry bookkeeping by Stokes integrals:

$$\oint \delta M + \oint \delta P = 0 \qquad (25.3.3)$$

The sum of the monetary circuit (δ M) and the production circuit (δ P) is zero, but each Stokes integral is not equal to zero.

## 25.4 The laws of economics in integral form

Eq. (3.3) may be written as

$$\oint \delta M = - \oint \delta P \qquad (25.4.1)$$

The monetary circuit measures the productive circuit. This is the basic law of-economics in integral form based on double-entry accounting.

### 25.4.1 The monetary circuit

The monetary balance is a closed Stokes integral:

$$\oint \delta M = \int_{In}^{H} \delta M_1 + \int_{H}^{In} \delta M_2 = Y_H - C_H = \Delta M_H. \qquad (25.4.2).$$

If the monetary Stokes integral is positive, the spiral goes up after each round by $\Delta M_H$. The term $\Delta M_H$ is the output of production of households. The value depends on the path of the integral, of the special way of production. If the monetary Stokes integral is negative, the spiral goes down after each round by $\Delta M_H$. Equation (25.4.2) is equivalent to eq. (25.2.1).

Income ($Y_H$) and costs ($C_H$) of households are part of a closed monetary circuit or integral: Along path (1) industry (In) pays income ($Y_H$) to households (H),

$$Y_H = \int_{In}^{H} \delta M_1 \qquad (25.4.3).$$

Along path (2) households (H) pay consumption costs ($C_H$) to industry (In),

$$C_H = - \int_{H}^{In} \delta M_2 \qquad (25.4.4)$$

Output, income and costs depend on the path of integration, on the way of production and cannot be calculated in advance. Output, income and costs are always related to a certain system, to a household, a company or an economy.

### 25.4.2 The productive circuit

The productive circuit - like the monetary balance in eq. (4.2) - is again a closed Stokes integral Labour and goods are Stokes line integrals of the same closed circuit.

$$\oint \delta P = \int_{In}^{H} \delta P_1 + \int_{H}^{In} \delta P_2 = L_H - G_H = \Delta P_H. \qquad (25.4.5).$$

Along path (1) industry (In) sends goods ($G_H$) to households (H),

$$G_H = \int_{In}^{H} \delta P_1 \qquad (25.4.6).$$

Along path (2) households (H) invest labour ($L_H$) at work in industry (In),

$$L_H = - \int_{H}^{In} \delta P_2 \qquad (25.4.7)$$

Output ($\Delta M_H$), income ($Y_H$), cost ($C_H$), labour ($L_H$) and goods ($G_H$) are the integral terms of calculus based economics. These terms correspond to terms in neoclassical theory, but calculus based theory clarifies the ex post character of these integral terms.

## 25.5 The laws of economics in differential forms

In chapter we have found the fundamental law of economics of money-based societies, eq.(25.4.1),

$$\oint \delta M = -\oint \delta P$$

The monetary circuit measures the production circuits in monetary units. This integral law is also the basis for the laws of economics in differential forms:

### 25.5.1 The first law of economics

The first law of economics in differential forms is

$$\delta\, M = d\, K - \delta\, P \qquad (25.5.1).$$

Integrating eq. (5.1) by a closed integral leads back to the fundamental law, eq. (5.8). The new term (d K) is an exact differential form and the closed integral of (d K) will be zero. The first law of economics contains three differential forms with the common dimension *money*:

$\delta$ M is inexact and refers to the *ex post* terms of income ($\delta$ Y), costs ($\delta$ C) and surplus or profits.
$\delta$ P is also inexact and refers to the *ex post* term of production, to goods (G) and labour (W). Production is not just the number of labourers, but real physical work.
d K is exact and (K) a function. The meaning of (d K) is capital, the fields of the farmer, the company of the entrepreneur, the industries of an economy, the earth of all beings. Capital of an economic system includes property, firms, houses and money in cash. We will come back to this in a separate chapter.

The first law is the balance of every economic system: *Profits ($\delta M$) depend on capital (d K) and labour ($\delta P$).* This result is well known in economics, but so far, it has not been stated in a proper mathematical form by a differential equation.

The first law corresponds to the statement by Adam Smith in "Wealth of Nations" (I.6.11): *"It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased; and its value, to those who possess it, and who want to exchange it for some new productions, is precisely equal to the quantity of labour which it can enable them to purchase or command."*

### 25.5.2 The second law of economics

According to calculus (chapter 2.8) a not exact differential form ($\delta$ M) may be transformed into an exact differential form (d F) by an integrating factor $\lambda$,

$$\delta\, M = \lambda\, d\, F \qquad (25.5.2).$$

We may call this the second law of economics. ($\delta$ M) is inexact or *ex post*. For positive values ($\delta$ M) refers to production output, profits or income ($\delta$ Y). At negative values ($\delta$ M) refers to losses or costs ($\delta$ C).
For positive values ($\delta$ M) may be replaced by income ($\delta$ Y),

$$\delta\, Y = \lambda\, d\, F \qquad (25.5.2\ a),$$

Eq. (5.2 a) replaces the erratic Solow equation Y = F (K, N) in eq.(1.1) due to the rules of calculus:

1. Complete or exact differential forms (d F) lead to a function (F) by a Riemann integral.
2. Incomplete or inexact differential forms ($\delta$ Y) do not have a stem function Y! The Stokes integral of not exact differentials ($\delta$ Y) is not a unique function, but depends on the path of integration. Income as an *ex post* term cannot be a function, it must be a not-exact differential form: ($\delta$ Y)!

### 25.5.3 The third law of economics

The second law replaces the inexact differential form ($\delta$ M) by an exact differential form (d F) and an integrating factor ($\lambda$). In the same way, we may replace the inexact form of production ($\delta$P) by the exact differential (d V) and the integrating factor (p),

$$\delta P = - p \, d V \qquad (25.5.3).$$

We have, presently, deducted eq. (5.3) formally, but this needs further discussion of the new economic terms (p) and (V). People go to work in the fields or in industry. Why do they work? The answer: Everybody gets periodically hungry. Hunger is an inner pressure (p) to obtain food. This is true for all creatures. Even in modern times, where nobody needs to be hungry, there is a public pressure (p) for everybody to work.
The function (V) corresponds to space, to the area, that we need for living, for agriculture, for industries. Pressure (p) or space (V) are two possible production factors that we may apply to economics.

### 25.5.4 Economics and thermodynamics

The differential laws of calculus based economic theory have the same mathematical structure as the laws of thermodynamics:

$$\delta Q = d E - \delta W \qquad (25.5.4).$$

In thermodynamics heat ($\delta$ Q) and work ($\delta$ W) are not-exact differential forms. The not-exact differential of heat ($\delta$ Q) may be linked to an exact differential form (d S) by an integrating factor (T):

$$\delta Q = T \, d S (E, W) \qquad (25.5.5).$$

Also eq. (25.5.3) corresponds to thermodynamics, where the not exact form of work ($\delta$ W) is replaced by an exact form of volume (d V) and an integrating factor of pressure (p):

$$\delta W = - p \, d V \qquad (25.5.6).$$

But this is not only a formal coincidence but rather an identity:
1. In eq.(4.5) labour is part of the closed cycle of production. In neoclassical economics labour (L) is interpreted as the number (N) of laborers. But 10 people standing around in a field will stay hungry unless they go to work and pick the crops. Labor may be measured in energy units and is equivalent to physical work (W), L = W!
2. The oil price is an international standard to transform capital (K) into energy (E) and vice versa. We may conclude: K = E!
3. Income ($\delta$Y) like capital is measured in monetary units. With the oil price income may be turned into a quantity ($\delta$ Q) measured in energy units, $\delta Y = \delta Q$.

Table I shows the corresponding functions in economics and thermodynamics:

| Symbol | Economics | Unit | | Symbol | Thermodynamics | Unit |
|---|---|---|---|---|---|---|
| **Y** | Income, Costs | €, $, £. ¥ | ↔ | **Q** | Heat | kWh |
| **K** | Capital | €, $, £. ¥ | ↔ | **E** | Energy | kWh |
| **L** | Production, Labour | €, $, £. ¥ | ↔ | **W** | Work | kWh |
| **λ** | Mean capital | €, $, £. ¥ | ↔ | **T** | Mean energy | kWh |
| **F** | Production function | - | ↔ | **S** | Entropy | - |
| **N** | Number | - | ↔ | **N** | Number | - |
| **p** | Pressure | | ↔ | **p** | Pressure | |
| **V** | Space | | ↔ | **V** | Volume | |

**Table 25.1** of corresponding functions of economics and thermodynamics.

Table 25.1 shows the corresponding functions of economics and thermodynamics resulting from the first and second laws. In addition we may compare thermodynamic pressure to economic or social pressure, and volume of materials to space of economic systems.

### 25.5.5 Standard of living as economic temperature

The integrating factor (λ) exists in all economic systems, in production, in markets and finance. In interacting economic systems and efficient markets we always find a common economic level (λ). A market will lead to a common price level (λ) for each commodity, an economy will have a common mean standard of living (λ). The integrating factor λ is the mean capital and equivalent to temperature (T) as the mean energy, T = λ.

This equivalence of (T) and (λ) is shown in fig. 1: GDP per capita and energy consumption per capita are equivalent for most of the 126 largest countries in the world. Both, mean capital and mean energy consumption follow the same line.

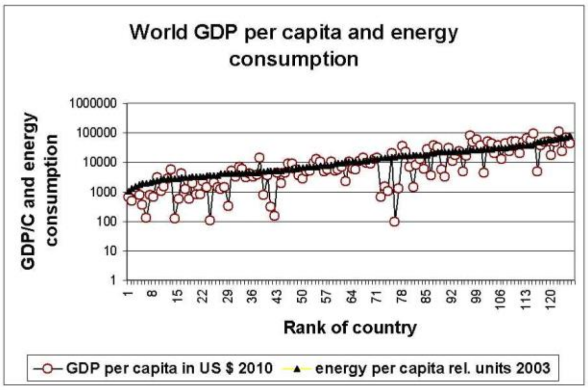


**Fig. 25.6** GDP per capita and energy consumption per capita follow the same line for most of the 126 largest countries in the world. Both, mean capital and mean energy consumption are equivalent. Economics and thermodynamics are corresponding.

The standard of living ($\lambda$) may be defined by the mean capital per capita

$$\lambda = K / c\, N. \tag{25.5.7}$$

It is proportional to the social temperature or energy consumption per capita

$$T = E / c\, N. \tag{25.5.8}$$

The constant c may be called specific income and reflects the degrees of fredom to obtain income from work, stocks etc. It corresponds to the specific heat or degrees of freedom in thermodynamics.

### 25.5.6 Capital

According to the first law capital (d K) is the only monetary term that is represented by an exact differential form (d K) and exists "ex ante". Capital corresponds to energy in thermodynamics.

The difference between capital (*ex ante*) and income (*ex post*) is clear for a boy that goes out to work at a restaurant. He has five Dollar in his pocket, which will be his capital tonight. But he does not know how much he will earn from tips at the restaurant. He will count his income afterwards. For the boy capital is the cash money in his pocket.

According to eq.(5.1) capital is also equivalent to all goods that do not lose in value ($\delta M = 0$):

$$\delta M = d K - \delta P = 0 \tag{25.5.9}$$

The capital of a farmer is given by the fields, they are the basis for production and profit. The capital of a company is the production plant, the machines and investments. The capital of countries are resources like water, air, land, oil. But capital is also knowledge, technology, industry, cities, houses, universities, education, infra structure.

### 25.5.7 Entropy as the new production function

A most important result is the equivalence of production function (F) and entropy (S), F = S. Entropy is linking thermodynamics to statistical mechanics,

$$S = \ln P(N) \tag{25.5.10}$$

P (N) is the probability of arrangement of the N elements (atoms) in the thermodynamic system. In a system with (i) different elements (atoms) the entropy may be calculated from the relative number is $x_i = N_i / N$ of atoms,

$$S = N \Sigma x_i \ln x_i \tag{25.5.11}$$

This is called the Shannon entropy.

Accordingly, the production function of an economic system of (i) different elements (goods, people, etc.) may be calculated from the relative number $x_i = N_i / N$ of people, goods, etc.

$$F = N \Sigma x_i \ln x_i \tag{25.5.12}$$

The function is shown in fig. 25.7:

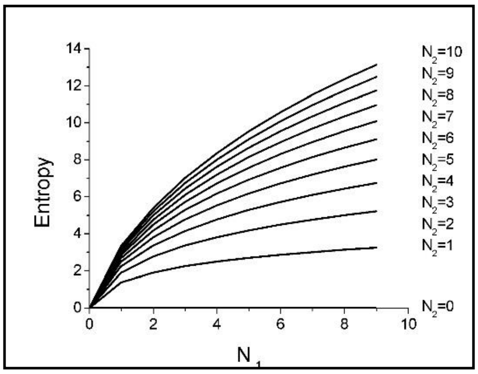


**Fig. 25.7** The entropy production function F ($N_1$, $N_2$) in eq.(9) is plotted versus $N_1$ in the range from 0 to 10. The parameter $N_2$ is in the range from 0 to 10.

Entropy replaces the Cobb Douglas production function of neoclassical economics

$$F = -N (x_i)^{\alpha} (x_j)^{ß} \qquad (25.5.13).$$

Both function are very similar, but according to figs 5.2 and 5.3 entropy is larger by a factor 1,4. Entropy is the natural and optimal production function.

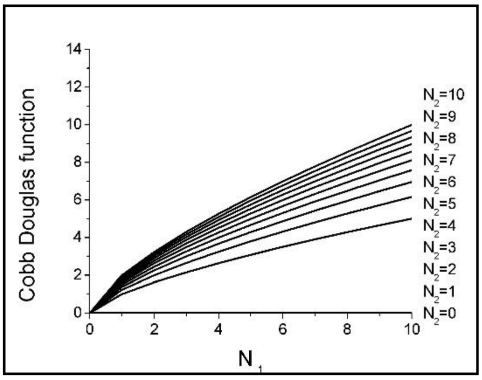


**Fig. 25.8** The Cobb Douglas production function F ($N_2$) = A $N_1^{\alpha}$ $N_2^{1-\alpha}$ in eq.(5.13) is plotted versus $N_1$ in the range from 0 to 10. The parameter is $N_2$ in the range from 0 to 10. The parameters are N = 1 and $\alpha$ = 0,7.

Indeed, entropy in fig. 25.7 is the natural production function, which is always more optimal than the Cobb Douglas function in fig. 25.8. In addition there is no elasticity in calculus based economics, which is used in standard economics to adjust to real data. Entropy needs no adjustment to data. In addition entropy is a function of the economic elements and leads to micro-economics and probability P (N) to statistical economics.

### 25.5.8 Entropy and work

The new production function entropy is a measure of disorder. This leads to a new understanding of entropy in thermodynamics and economics:

*1. Thermodynamics: Combining the first and second law of thermodynamics we obtain*

$$T\,d\,S. = d\,E - \delta\,W \qquad (25.5.14).$$

*A light breeze in a park with the energy (d E) will easily empty a paper basket and generate more and more disorder (T d S > 0). The paper will distribute throughout the park and never come back into the basket. Positive entropy means creating disorder or distributing items.*
*But a janitor may work (δ W) and sweep the paper together and put it back into the basket.*
*Work reduces disorder: (T d S < 0). Negative entropy means reducing disorder or ordering, collecting items.*

*2. Economics: In the same way we may combine the first and second law of economics,*

$$\lambda\,d\,F = d\,K - \delta\,P \qquad (25.5.15).$$

*The capital (d K) you pay for a snack will easily empty your purse and distribute (λ d F > 0) the money to the shop keeper. The money will never come back into the purse. But in the afternoon you may work (δ P) in the office and the money will come back into your purse.*

We may solve eq. (5.15) for (δ P):

$$\delta\,P = d\,K - \lambda\,d\,F \qquad (25.5.16).$$

Labor (d P) increases capital (d K) and reduces disorder (– λ d F). Work means ordering:

**Fig. 25.9** Production of automobiles requires the ordering of many parts: screws, nuts and bolts, wires, tires, wheels etc. All parts have to be placed in the correct position and in the right order, hence entropy d F < 0.

Entropy reduction also applies to mental work: Mental work orders ideas like in a puzzle:

Brain work: g+i+r+r+n+o+d+e → ordering

Medical doctors order deficiencies within a body, teachers order or develop the minds of young people. Housewives have known for long times that making order is hard (unpaid) work! This may be one reason that today most women prefer to work as professionals outside of the house.

### 25.5.9 Production costs

According to eq. (5.16) production costs (δ P) depends on capital or energy costs (d K), on the amount of ordering (– d F), and on the standard of living (λ) of the producing country. Many companies produce cars with the same energy costs and parts in the same order (d F) in Europe and China. However, wages are much lower in China than in Europe due to the lower standard of living (λ) in China.

## 25.6 Capitalism

In mathematics the closed production circuit is a Stokes integral. In physics this is called the Carnot cycle of motors. In economics it is the mechanism of capitalism.

### 25.6.1 The Carnot production cycle of capitalism

We now come back to the productive and monetary circuits in chapter 25.3. The equivalence of productive and monetary circuits, eq. (25.4.8), and the second law, eq. (25.5.2), which replaces profits ($\delta\ M$) by ($\lambda$ d F), lead to the fundamental equation of production and trade:

$$-\oint \delta P = \oint \delta M = \oint \lambda\, d\, F \qquad (25.6.1).$$

Productive and monetary circuits are now depending on the production factors ($\lambda$) and (F), and we can give the productive and monetary circuits proper coordinates:
The factor ($\lambda$) is the Y – axis and represents the value or price. The entropy (F) is the X - axis: (+d F) corresponds to distributing or selling products and (- d F) to collecting or buying products. The integrals of eq. (6.1) are not-exact and depend on the path of integration. Carnot proposed an ideal path of integration, which was first used to explain the steam engine: integration along (F) at constant ($\lambda$) and then integration along ($\lambda$) at constant (F). In this way we obtain four different lines of the line integrals eq. (25.6.1) in fig. 25.10.

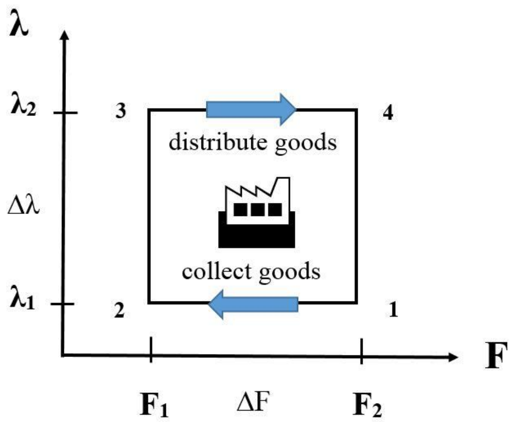


**Fig. 25.10.** The mechanism of capitalism is the Carnot production process of industry: produce cheap and sell expensive! The Carnot production process is an ideal process originally used to explain a steam engine.
According to Carnot, the production process is divided into four sections:
*1 – 2:* Workers of a company collect or assemble (-d F) industrial goods at low price ($\lambda_1$);
*2 – 3:* Products are refined to create a higher value (d $\lambda$) or are transported (exported) to a region, where the products are more valuable ($\lambda_2$);
*3 – 4:* Products are distributed (+d F) and sold to customers at higher price ($\lambda_2$);
*4 – 1:* After longer time of use the products will break and must be transported at lower value ($\lambda_1$) to centers of recycling. Without recycling the process starts directly again at point (1).
The Carnot production process is a two level mechanism at two price levels, $\lambda_2$ and $\lambda_1$.

*Example 25.6.1: The production circuit of European import of clothes from Bangladesh*
The details of the clothes production circuit are derived from the law of production, eq. (5.16),

$$\delta P = c\, N\, d\lambda \; - \lambda\, d F \tag{25.6.2}$$

*1 – 2:* A Bangladesh export company collects clothes (-d F) produced by workers (seamstresses) at low costs ($\lambda_1$), and brought to the airport of Dhaka.

$$\delta P_1 = \lambda_1 (- d F) \tag{25.6.3}$$

($P_1$) is the value of labour input and depends on the amount of seaming (-d F) and of the very low standard of living of the workers ($\lambda_1$) in Bangladesh. This corresponds to an isothermal process in thermodynamics.
*2 – 3:* Clothes are flown to Frankfurt in Europe, where clothes has a higher value ($\lambda_2$):

$$\delta P_2 = c\, N\, d\lambda \tag{25.6.4}$$

During transport, export by car, train ship or plain the value ($\lambda$) of clothes rises, but the distribution or entropy (F) is constant. This is called an adiabatic process in thermodynamics.
*3 – 4:* Clothes are distributed and sold to customers in Europe at a higher price ($\lambda_2$),

$$\delta P_3 = \lambda_2\, d F \tag{25.6.5}$$

Within one season the price ($\lambda_2$) stays constant, this is again an *isothermal* process in thermodynamics.
*4 – 1:* In time the clothes become old fashioned, or may break. They are transported to recycling centers. The value of the used clothes has declined ($d\lambda$),

$$\delta P_4 = - c\, N\, d\lambda \tag{25.6.6}$$

During transport e.g. in a container the products stay together, this means constant entropy (F), transport is again adiabatic. Without recycling the last step may be omitted. The new Carnot cycle of capitalism starts again at point (1).

*Example 25.6.2: The production cycle of automobiles in a factory*
*1 – 2:* The workers of an automobile factory are collecting**,** ordering ($d F < 0$) and putting together a large number of parts to construct the automobile at constant wages ($\lambda_1$).
*2 – 3:* The automobile company transports (exports/imports) the produced cars from the factory to the car dealer, where cars may be sold at a higher price ($\lambda_2$). During transport by truck, train or ship, the cars stay together, the entropy ($F_1$) does not change.
*3 – 4:* The car dealer distributes or sells ($d F > 0$) the cars to customers at a high price ($\lambda_2$).
*4 – 1:* After years of use cars will break or rust and must be transported to a recycling center for a low price ($\lambda_1$).

*Example 25.6.3: The production cycle of potatoes at a farm*
*1 – 2:* On a farm workers collect ($d F < 0$) potatoes in the fields at constant low wage ($\lambda_1$).
*2 – 3:* The farmer transports (exports/imports) the collected cheap potatoes ($\lambda_1$) from the fields to the market, where potatoes may be sold at a higher price ($\lambda_2$). Since the collected potatoes stay in the basket during the transport, the entropy ($F_1$) does not change.
*3 – 4:* At the market the farmer distributes or sells ($d F > 0$) the potatoes to customers at a constant high price ($\lambda_2$).
*4 – 1:* As the next year (cycle) starts with new potatoes, recycling may be omitted.

### 25.6.2 The monetary cycle of capitalism

The monetary circuit (δ M) runs opposite to the production circuit (δ M), eq. 9.1:

$$-\oint \delta P = \oint \delta M = \oint \lambda \, d \, F \qquad (25.6.7).$$

In the productive circuit in fig. 9.1 goods, commodities, manufactured products are collected from workers and distributed to customers. In the monetary circuit of fig. 25.11 money is collected from customers and distributed to workers:

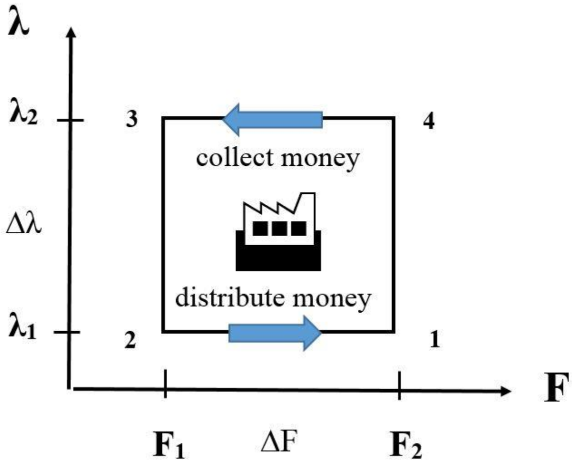


**Fig. 25.11** The monetary circuit of capitalism in industry according to Carnot: Earn (collect) much and pay (distribute) little! The monetary circuit runs opposite to production.

The monetary circuit is again divided into four sections:

*4 – 3:* A company outlet store collects (-d F) money from customers for manufactured products. During one season the high price ($\lambda_2$) of the products stays constant. This money is the income (Y) of the company.

*3 – 2:* The company outlet store sends money for manufactured products to the financial center of the company, transporting the money in a safe is a process of constant entropy ($F_1$).

*2 – 1:* The low wages ($\lambda_1$), which are distributed (d F) to workers, are the labour costs (C) of the company.

*1 – 4:* The production company pays for materials to be recycled. Without recycling the Carnot process starts again directly at point (1).

*4-3-2-1-4:* The area of the Carnot cycle, $Y_{Ind} - C_{Ind} = \Delta M_{Ind}$ is the profit of the company,

$$\oint \delta M = \oint \lambda \, d \, F = Y_F - C_F = (\lambda_2 - \lambda_1) \, \Delta F \qquad (25.6.8).$$

Again, the Carnot production process is a two level mechanism at two price levels, $\lambda_2$ and $\lambda_1$.

*Example 25.6.4: The monetary circuit of European import of clothes from Bangladesh*

*4 – 3:* The importer of clothes collects money (- d F) from European customers for Bangladesh clothes at constant high price ($\lambda_2$). This money is the income ($Y_2$) of the import company,

$$Y_2 = \lambda_2 \, \Delta F \qquad (25.6.9)$$

Within the same season the price level ($\lambda_2$) is constant, this is called an isothermal process in thermodynamics.

*3 – 2:* Importers in Europe send money for clothes to the exporting company in Bangladesh, transporting (money) is again an adiabatic process, $\Delta F = 0$,

$$\Delta M_1 = \lambda \Delta F = 0 \qquad (25.6.10)$$

*2 – 1:* Industry pays (d F) workers for their products a constant low price ($\lambda_1$). Labor costs of industry are the incomes of the workers.

$$C_2 = \lambda_2 \Delta F \qquad (25.6.11)$$

*1 – 4:* Industry pays recycling centers for materials that may be recycled in the next production circuit.

$$\Delta M_2 = \lambda \Delta F = 0 \qquad (25.6.12)$$

*4-3-2-1-4:* The area of the Carnot cycle, $Y_{Ind} – C_{Ind} = \Delta M_{Ind}$ is the profit of the company.
*Example 9.5: The monetary cycle on a potato farm*
*4 – 3:* At the market the potato farmer collects money ($d F < 0$) a constant high price ($\lambda_2$) from the customers.
*3 – 2:* The farmer transports the collected money in his pocket at constant low entropy ($F_1$) from the market to the fields, transport (export/import) is again an adiabatic process.
*2 – 1:* In the fields the farmer distributes constant low wage ($\lambda_1$) to the workers.
*1 – 4:* The farmer pays for fertilizer.
*4-3-2-1-4:* The area of the Carnot cycle, $Y_{Ind} – C_{Ind} = \Delta M_{Ind}$ is the profit of the farmer.

**25.6.3 Production and trade**

Production and trade are closely linked together. In productive companies the products are produced cheaply ($\lambda_1$) and sold at a higher price ($\lambda_2$). In trade oriented companies products are bought cheaply ($\lambda_1$) and sold more expensively ($\lambda_2$). Both follow the Carnot mechanism: money and goods have to be traded at two different price levels, $\lambda_2$ and $\lambda_1$!
At one levels, ($\lambda_1$), the exchange of money and goods does not change the total capital of buyers or sellers, as demonstrated in double entry bookkeeping. This may be expressed by the first law

$$\delta P + \delta M = d K = 0 \qquad (25.6.13)$$

The action of buying or selling is an *iso-capital* process, and corresponds to an *iso-thermal* process in thermodynamics. Nobody can get rich by a single exchange of money and goods, unless he cheats with false money or manipulated goods. A profit is only possible by a second exchange of money and goods at a new level ($\lambda_2$)!

Trade like production is a Carnot process, the exchange of money and goods at two different price levels, the simultaneous combination of a productive and a monetary circuit. The profit according to eq. (6.1) is:

$$\oint \delta M = \oint \lambda \, d F = Y_F – C_F = (\lambda_2 - \lambda_1) \Delta F \qquad (25.6.14).$$

The profit of trade and production is the difference between incomes ($Y_F$) and costs ($C_F$). Profit grows with the amount of ordering $\Delta F$ and the improvement of the value $\Delta \lambda$ of products. The difference in value ($\lambda_2 - \lambda_1$) may be obtained in many ways:

1. The product may be refined by the workers: metals may be gilded or shaped into a new form, potatoes may be cooked in a restaurant, threads may be woven into a garment, etc.

2. Another way of raising the value of a product is exporting the product to a new location, where the product has a higher value.

3. A third way is waiting for a certain time period, until the product is more valuable.

**25.6.4 The Carnot process of production and trade is a two level mechanism: $\lambda_2$ and $\lambda_1$**
The Carnot production mechanism applies to all economic activities, to labour of households, to farmers, to production in industrial plants, to import of commodities from China, coal from South Africa or the financial activities of a savings bank. In thermodynamics the Carnot mechanism applies to heat engines like motors, generators, heat pumps or refrigerators.

Production always requires two separate constant levels $\lambda_1$ and $\lambda_2$ or classes, and vice versa, production always generates a two class society: in companies we have capital and labor, in markets we have buyers and sellers, in societies we find rich and poor, in the world economies we have first and third world counties. In physics a motor always requires two temperatures, hot and cold. The Carnot process in factories and in motors runs on the same fuel: oil.

**25.6.5 Efficiency**
The efficiency $\eta$ of the Carnot process is determined by

$$\eta = (\lambda_2 - \lambda_1) / \lambda_2 \qquad (25.6.15).$$

The efficiency grows with the difference in levels of ($\lambda$). The higher the difference ($\lambda_2 - \lambda_1$), the more efficient is the process. A cold motor does not run, only after a few cycles of heat production the inside of the motor will become hotter and the efficiency rises, if the outside is cooled down by air or water. This efficiency applies to motors as well as to economies.

**6.6 Scissor effect**
In most economies we observe a growing difference between rich and poor. This is sometimes called the scissor effect, fig. 25.12. This corresponds to thermodynamics: As soon as a refrigerator is turned on, the inside will get colder and the outside will get warmer. And as soon as a motor has started the heat produced during a cycle, is distributed to the inside and the motor will get hotter and more efficient.

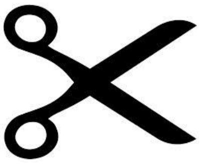

**Fig. 25.12** The growing difference between rich and poor is due to the growing efficiency of the Carnot production process. This is often called the scissor effect. The mechanism corresponds to the growing temperature of a motor or the falling temperature of a refrigerator.

**25.6.6 Efficiency and socio-economic models**
Efficiency is the basis of socio-economic programs of political parties and states:
**Capitalism:** Capital favors a high efficiency, $\eta \rightarrow$ **max!** This means high prices and low wages, which leads to a strong economy, and according to the scissor effect to a rising gap between rich and poor. A good example in Europe is Germany. In order to avoid aggressions between high and low income classes, the government of a strong economy can level out differences in incomes by taxes and by support of unemployed and other problem groups.
**Socialism (Labour):** Labour favors a lower efficiency, $\eta \rightarrow$ **min!** This means low prices and high wages. This leads to a weaker economy and a slowdown of the scissor effect between rich and poor. A good example in Europe seems to be France. The lower income classes still have a rather good standard of living, but the state cannot raise enough taxes to support problem groups.

**Communism:** Communism calls for a one class society in which the capital is owned by the workers, the proletarians. In a one class society ($\lambda_2 = \lambda_1 = \lambda_0$) the efficiency will be zero, $\eta \to 0$**!** This has been observed for all communist states, and has led to the downfall of all communist regimes in Europe.

In order to make a refrigerator work we have to close the door, inside and outside have to be separated. In the same way rich and poor classes have to be separated to make the Carnot production process work.

### 25.6.7 Economic growth of countries

Capital growth of a country may be calculated from capital as a function of standard of living and the population:

$$K = c\,N\,\lambda \qquad (25.6.16)$$

$$d\,K = c\,N\,d\,\lambda + \lambda\,c\,d\,N \qquad (25.6.17)$$

In every country the growth of capital is given by a change in standard of living and a change in population. We will discuss four different cases:

a) At constant capital the standard of living will drop with rising population.
b) At constant standard of living the population will rise.
c) At constant population the standard of living will rise.
d) A rising capital with rising population and rising standard of living is rarely observed.

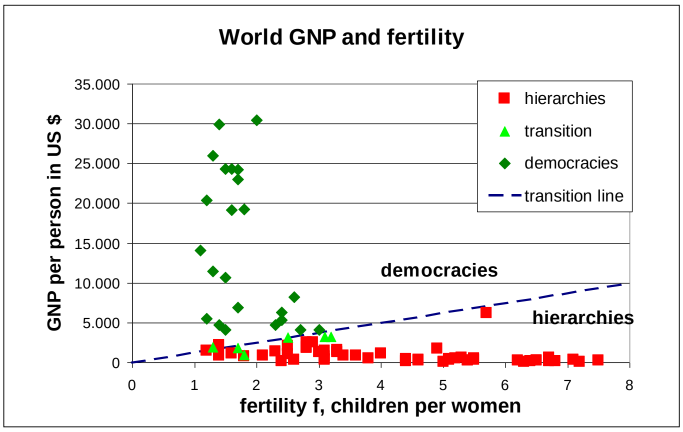


**Fig. 25.13** GNP and fertility for the 90 biggest countries in the world.

Figure 25.13 demonstrates that people either invest in pension funds or in children. People in rich and democratic countries have a high standard of living and low fertility. People in poor and not democratic countries have a low standard of living and a high fertility.
This distribution may only be changed by reducing fertility in poor countries. In many countries like Egypt economic growth being is eaten up by the growing population. Countries like Kerala in India have managed to reduce fertility drastically by building hospitals with controlled birth and by investing in the internet for jobs in rural areas. Similar programs must be installed in all poor areas to raise local economic growth and reduce population growth.

**25.7. Conclusion**

Calculus based economic theory leads to economic laws that are identical to those in thermodynamics. The new economic theory defines many terms of neoclassical economics in differential forms refering to economic properties like “ex ante” and “ex post”. Experimental data on “GDP and energy consumption” support the result that economic output depends on energy (E) and physical labour (W) rather than on capital (K) and the labour force (N).

The theory may be expanded further to economic growth, to microeconomics and finance using the new production function entropy as a gate to probability and statistics. This field is called econophysics and has been widely investigated in recent years.

Another field is the application of thermodynamics to social sciences. National or religious groups are heterogeneous many agent systems, which may be treated by the laws of heterogeneous many particle systems like alloys or other materials. These results have been discussed elsewhere [12 – 16].

**References**


[1] R. N. Mantegna, H. E. Stanley, *Introduction to Econophysics. Correlations and Complexity in Finance*. Cambridge University Press, Cambridge (USA) 2000.

[2] J. Mimkes, A thermodynamic formulation of economics in *Econophysics & Sociophysics*: Trends & Perspectives Bikas K. Chakrabarti, Anirban Chakraborti, Arnab Chatterjee (Eds.) WILEY-VCH Verlag, Weinheim, Germany (2006)

[3] Y. ARUKA and J. MIMKES, An Evolutionary Theory of Economic Interaction— Introduction to Socio- and Econo-Physics Evol. Inst. Econ. Rev. 2(2): 145–160 (2006)

[4] J. Mimkes, Concepts of Thermodynamics in Economic Growth In: *The complex Networks in Economic Interactions* Akira Namatame, Taisei Kaizouji, Yuji Aruka, Ed. Lecture Notes in Economics and Mathematical Systems Springer, Berlin, Heidelberg (2006)

[5] V. M. Yakovenko and J. B. Rosser, *Colloquium: Statistical Mechanics of Money, Wealth and Income,* Rev. Mod. Phys. 81, 1703 (2009).

[6] J. Mimkes, *Putty and Clay - Calculus and Neoclassical Theory* Dynamics of Socio-Economic Systems, Vol.2 Number 1: 1-8. (2010).

[7] J. Mimkes, *Stokes integral of economic growth: Calculus and the Solow model* Physica A 389 1665-1676 (2010)

[8] J. Mimkes, *Introduction to Macro-Econophysics and Finance* Continuum Mechanics and Thermodynamics, Springer, Volume 24, Issue 4-6, pp 731-737, (November 2012)

[9] P. Richmond, J. Mimkes, S. Hutzler, *Econophysics and Physical Economics*, Oxford University Press (Sept. 2013).

[10] Cobb, C. W. and Douglas, P. H. : „A Theory of Production“ in American Economic Review, Mar 28 Supplement, 18 (1928) 139-165

[11] Solow, A R. M., Contribution to the Theory of Economic Growth, The Quarterly Journal of Economics, 70 (Feb. 1956) 65-94

[12] J. Mimkes “Binary Alloys as a Model for the Multicultural Society” J. Thermal Anal. 43 (1995) 521

[13] J. Mimkes “Society as Many Particle System” J. Thermal Anal. Calor. 60 (2000) 1055

[14] J. Šesták: ”Thermodynamics and society – laws versus feelings”, Chapter 18 in book “Heat, Thermal Analysis and Society” by Nucleus, Hradec Kralove, Czechia (ISBN 8-86225-54-2), pp.298-302. [2004]

[15] J. Šesták: “Thermodynamics, econophysics and societal behavior”, Chapter 8 in book „Science of Heat and Thermophysical Studies: a generalized approach to thermal analysis“ by Elsevier, Amsterdam, Netherlands (ISBN 444 51954 8) pp. 230-246 (2005)

[16] J. Mimkes, A thermodynamic formulation of social science in *Econophysics & Sociophysics*: Trends & Perspectives Bikas K. Chakrabarti, Anirban Chakraborti, Arnab Chatterjee (Eds.) WILEY-VCH Verlag, Weinheim, Germany (2006)