Kohärente und mesoskopische Systeme

Electron or hole transport in commercially available electronic devices is governed by various scattering mechanisms. On the other hand, devices based on coherent transport (transport without scattering) are still at the developmental stage. To understand the coherent length or dephasing length, let us first consider an electron that undergoes an elastic collision, where the initial, $\psi_{\mathrm{i}}(\boldsymbol{r}, t)$, and final, $\psi_{\mathrm{f}}(\boldsymbol{r}, t)$, wave functions are (Mitin *et al.*)

$$\psi_{\mathrm{i}}(\boldsymbol{r}, t) = \mathrm{e}^{-i\omega t}\mathrm{e}^{i\boldsymbol{k}.\boldsymbol{r}} \text{and} \psi_{\mathrm{f}}(\boldsymbol{r}, t) = \mathrm{e}^{-i\omega t} \sum_{\boldsymbol{k}', \boldsymbol{k}'=\boldsymbol{k}} A_{\boldsymbol{k}'} e^{i\boldsymbol{k}'.\boldsymbol{r}} = \mathrm{e}^{-i\omega t}\psi(\boldsymbol{r}),$$

where $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the wave vectors before and after the scattering event. For elastic scattering, we have $\boldsymbol{k} = \boldsymbol{k}'$, which means that the momentum is conserved, and $|A_{\boldsymbol{k}'}|^2$ is the probability of finding the electron with a wave vector $\boldsymbol{k}'$ after scattering. From Equation 5.179, one can obtain $|\psi_{\mathrm{f}}(\boldsymbol{r}, t)|^2 = |\psi(\boldsymbol{r})|^2$, which means that the spatial distribution remains independent of time after scattering.

Kohärente und mesoskopische Systeme

For inelastic scattering, the electron wave function after scattering has different energies and time dependencies according to the following:

$$\psi_i(\boldsymbol{r},t) = e^{-i\omega(\boldsymbol{k})t} e^{i\boldsymbol{k}.\boldsymbol{r}} \quad \text{and} \psi_f(\boldsymbol{r},t) = \sum_{\boldsymbol{k}',\boldsymbol{k}'\neq\boldsymbol{k}} A_{k'} e^{-i\omega(\boldsymbol{k}')t} e^{i\boldsymbol{k}'.\boldsymbol{r}}.$$

The time-dependent component of the wave function of the scattered electron cannot be factored out of the sum, and $|\psi_f(\boldsymbol{r},t)|^2$ is now a function of time. For inelastic scattering, the electron preserves its quantum coherence for a distance equal to or less than the inelastic scattering length, l_i. In general, l_i is larger than the de Broglie wavelength. A comparison between various characteristic lengths as compared to the de Broglie wavelength is shown in Fig.

Chrakteristische Längen

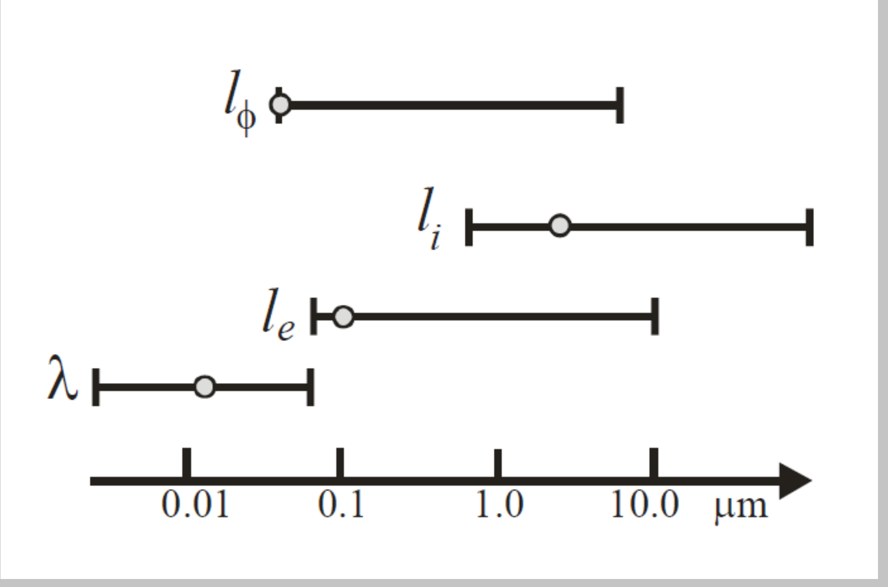

FIGURE 5.25 Intervals for the characteristic lengths: λ, de Broglie wavelength; l_e, mean free path; l_i, inelastic scattering length; and l_φ, coherence length in semiconductor materials. As an example, the lengths are marked by circles for Si at $T = 77$ K, assuming that the electron mobility equals 10^4 cm^2/V/s(after Mitinet *et al*.).

Dephasierungslänge

The electrons' dephasing length or coherent length, l_φ, is the distance that the electrons travel before losing their quantum mechanical coherence, which is a result of the large spreading of the wave function phases. The dephasing effect is caused by inelastic collisions, temperature spreading of phases, or both, which leads to the assumption that the dephasing length is determined by the smaller value of the inelastic length or the thermal diffusion length. The dephasing length, l_φ, is thus the distance that the electron transport has quantum characteristics. Systems in which electrons maintain coherence and remain in phase during transport are called *mesoscopic systems*, which have properties strongly depending on the geometry of the sample, contacts, and quantum structures.

Zener - Bloch - Oszillation

One of the simplest examples of quantum transport is electron transport in the absence of any scattering. A system with no scattering is a perfect crystalline solid in which the equation of motion of the electron is

$$\frac{dp}{dt} = \hbar\frac{dk}{dt} = e\mathcal{E}.$$

The electron starts at the bottom of the energy band and moves along the E versus k curve until it reaches the Brillouin zone edge. Since we have a perfect crystal, the energy bands are periodic in the k-space. Thus, when the electron reaches the zone edge, it is reflected and starts to lose its energy and then continues the cycle under the influence of the electric field. The momentum of the electron changes direction as the electron passes through the zone edges, leading to oscillations in k-space (and consequently in the real space). These oscillations are called *Zener–Bolch oscillations*, and their frequency is given by

$$\omega = \frac{ae\mathcal{E}}{\hbar} = \frac{ae\mathcal{E}}{\hbar},$$

where a is the lattice constant. For an electric field of the order of 10^7 V/m, the frequency of the Zener–Bloch oscillations is $\sim 1.21 \times 10^{12}$ Hz. This frequency range is very important for high speed devices.

Tunneln durch einfache Barriere

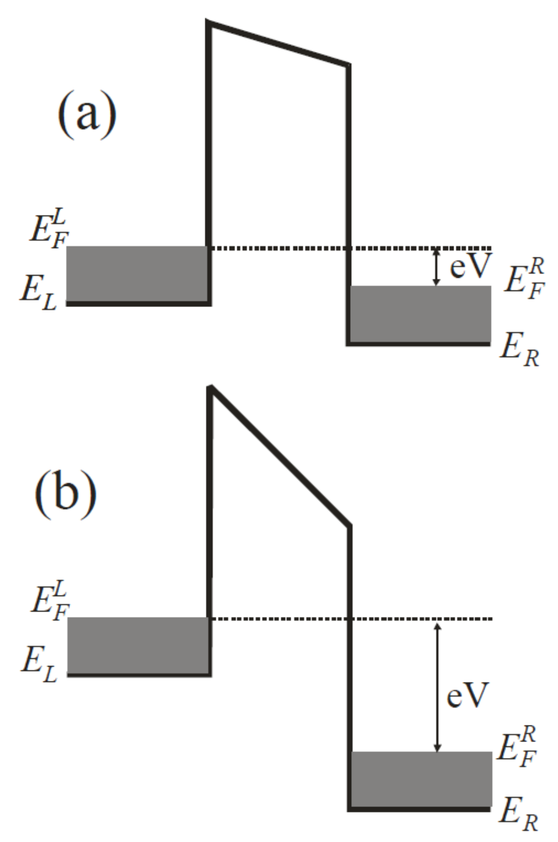

FIGURE 5.26 A single barrier with Fermi electron sea on both sides is shown for (a) small bias and (b) large bias.

Tunneln durch einfache Barriere

An expression for the conductance of a system in which the phase coherence is maintained can be derived for different contacts and sample geometries. Consider a one-dimensional simple barrier under bias voltage, as shown in Fig. 5.26. Under a small bias voltage, as shown in Fig. 5.26a, the electrons tunnel from both left to right and right to left. Under a bias voltage, each side of the barrier has its own Fermi energy level with a difference of $E_F^L - E_F^R = eV$. As the bias voltage is increased, as shown in Fig. 5.26b, the electron tunneling from right to left becomes negligible.

The electric current depends on the tunneling transmission coefficient and is given by

$$I_L = \frac{2e}{2\pi} \int_{E_L}^{\infty} f(E, E_F^L) \upsilon(k) T(k) \mathrm{d}k = \frac{2e}{h} \int_{E_L}^{\infty} f(E, E_F^L) T(E) \mathrm{d}E,$$

$$I_R = -\frac{2e}{2\pi} \int_{E_R}^{\infty} f(E, E_F^R) \upsilon(k) T(k) \mathrm{d}k = -\frac{2e}{h} \int_{E_R}^{\infty} f(E, E_F^R) T(E) \mathrm{d}E$$

Tunneln durch einfache Barriere

$$I = I_L + L_R = \frac{2e}{h}\int_{E_L}^{\infty}\{f(E, E_F^L) - f(E, E_F^R)\}T(E)\mathrm{d}E,$$

$$f(E, E_F^L) - f(E, E_F^R) = -eV\frac{\partial f(E, E_F)}{\partial E},$$

$E_F^L = E_F + \frac{1}{2}eV$ and $E_R^L = E_F - \frac{1}{2}eV$.

$$I = \frac{2e^2V}{h}\int_{E_L}^{\infty}\left\{-\frac{\partial f(E, E_F)}{\partial E}\right\}T(E)\mathrm{d}E.$$

the conductance, G, as

$$G = \frac{I}{V} = \frac{2e^2}{h}\int_{E_L}^{\infty}\left\{-\frac{\partial f(E, E_F)}{\partial E}\right\}T(E)\,\mathrm{d}E.$$

$-\frac{\partial f(E,E_F)}{\partial E} \approx \delta(E - E_F)$,

$$G = \frac{2e^2}{h}\int_{E_L}^{\infty}\delta(E - E_F)T(E)\mathrm{d}E$$

Tunneln durch einfache Barriere

$$G = \frac{2e^2}{h}\int_{E_L}^{\infty} \delta(E - E_F)T(E)\mathrm{d}E \qquad = \frac{2e^2}{h}T(E_F).$$

The factor e²/h is known as the *quantum unit of conductance* and the corresponding resistance is $R = h/e^2 \approx 25.829\ \mathrm{k}\Omega$. Equation shows that the conductance is independent of the length of the sample, and depends solely on the transmission coefficient. For $T(E_F)1$, the conductance is $2e^2/h$, which is independent of the sample geometry. For higher temperatures, the above δ-function approximation is no longer valid and the integration of Equation 5.188 should be performed.

The conductance result expressed in Equation is very simplistic since it is derived for only one mode or one path that the electron will take when traveling from one contact to another through the sample. In reality, one has to sum the electron contributions from all different paths the electron can take as it moves from one contact to another. For many different paths or propagating states, Equation can be written as

$$G = \frac{2e^2}{h}\sum_{n,m} T(E_F, m, n) = 2G_0 \sum_{m,n} T(E_F, m, n),$$

where $G_0 = e^2/h$ and the sum is over all electron states, m and n, with energy $E < E_F$. Equation is called the *Landauer formula*. Each channel or mode has two quantum numbers, m and n, where, for example, m represents the mode or state of the electron when leaving the left contact and n represents the mode or state of the electron when arriving at the right contact, as illustrated in Fig. 5.27.

Köhärenter Transport durch Bauelement

Landauer formalism provides a means to understand the transport in terms of scattering process, as illustrated in Fig. 5.27, where, for mesoscopic structures, the electron waves can flow from one contact to maintain phase coherence. The phase coherence is maintained at low temperatures at which scattering processes due to phonons are suppressed. Thus, Landauer formalism is valid only at low temperatures and small bias voltages. An important property of phase coherence transport is the fluctuation observed in the conductivity (resistivity) as a function of magnetic field. In Equation 5.190, the sum is over the electron contribution

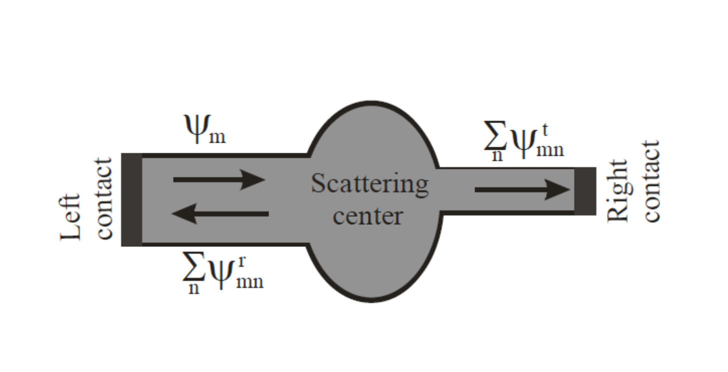

FIGURE 5.27 Illustration of coherent transport through a device with two leads. Each contact has many propagating states.

Leitwert als Funktion der Gatespannung

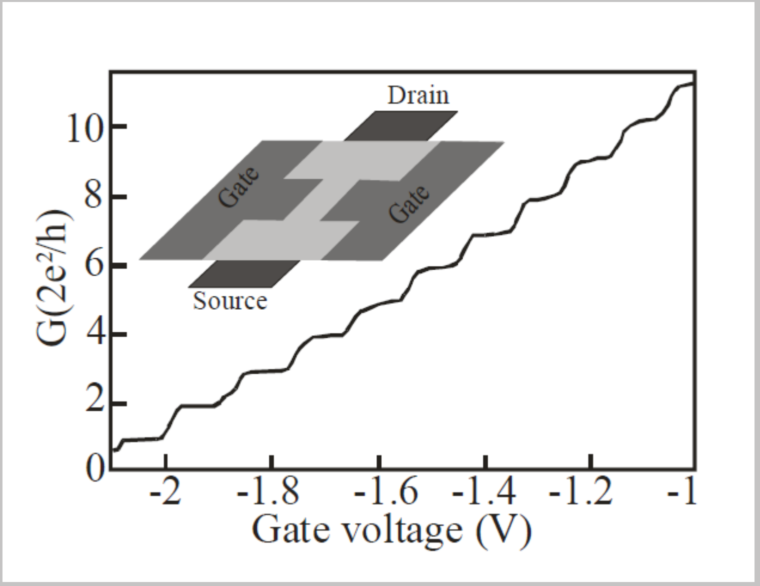

FIGURE 5.28 Conductance as a function of the gate voltage is plotted for GaAs/AlGaAs high electron mobility transistor (after van Wees *et al.*). The inset is a sketch of the MODFET showing the slit gate, drain, and source.

4-Spitzen Messmethode

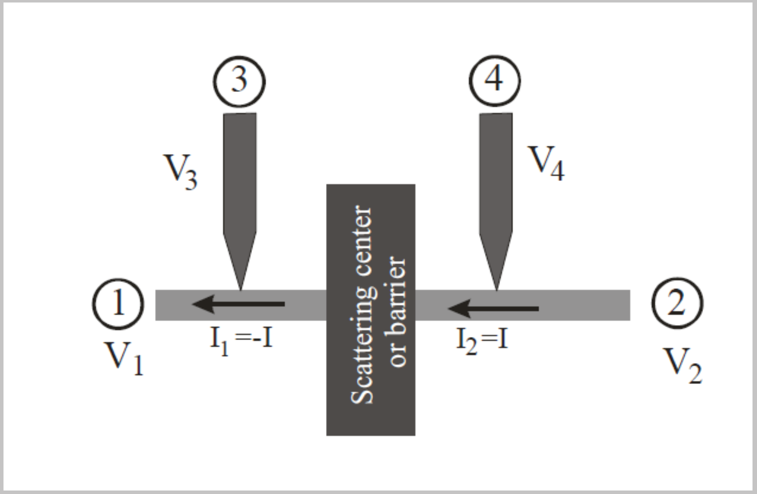

FIGURE 5.29 Four-probe measurements of the conductance of a scattering center (tunneling barrier) showing the four terminals at which the current and voltages can be measured.

The conductance is discussed briefly for a mesoscopic system of two leads and one electron path and for a system with two contacts and many electron paths. For a mesoscopic system with four contacts or probes, as shown in Fig. 7.29, the conductance can be derived as (Singh 2003 and Davies 1998)

$$G_{4\text{-probe}} = \frac{2e^2}{h}\frac{T}{R} = \frac{2e^2}{h}\frac{T}{1-T},$$

where T is the transmission coefficient and R is the reflection coefficient. Recall that $T+R=1$. It appears that there is a difference in the conductance obtained from two probes and four probes. For a weak transmitting barrier, there is a small difference between the conductance obtained from two probes and that obtained from four probes. But when the barrier is transparent or the transmission coefficient is approaching unity, the conductance expressed in Equation 5.191 approaches infinity, while the conductance obtained for two probes and expressed in Equation 5.189 takes the value $2e^2/h$. This behavior may be explained as follows: for a system in which the scattering center or the barrier is absent, the distribution of the electrons should be the same everywhere within the channel such that the voltage probe, in the case of a four-probe experiment, should read the same value at any point. Thus, the voltage difference between any two points is zero, giving rise to an infinite value for the conductance. When a bias voltage is applied to the two-probe configuration, an extra voltage appears because of an extra contact resistance of $h/(2e^2)$ in series with the sample. The extra resistance exists, even though the electrons are transmitted without any scattering.