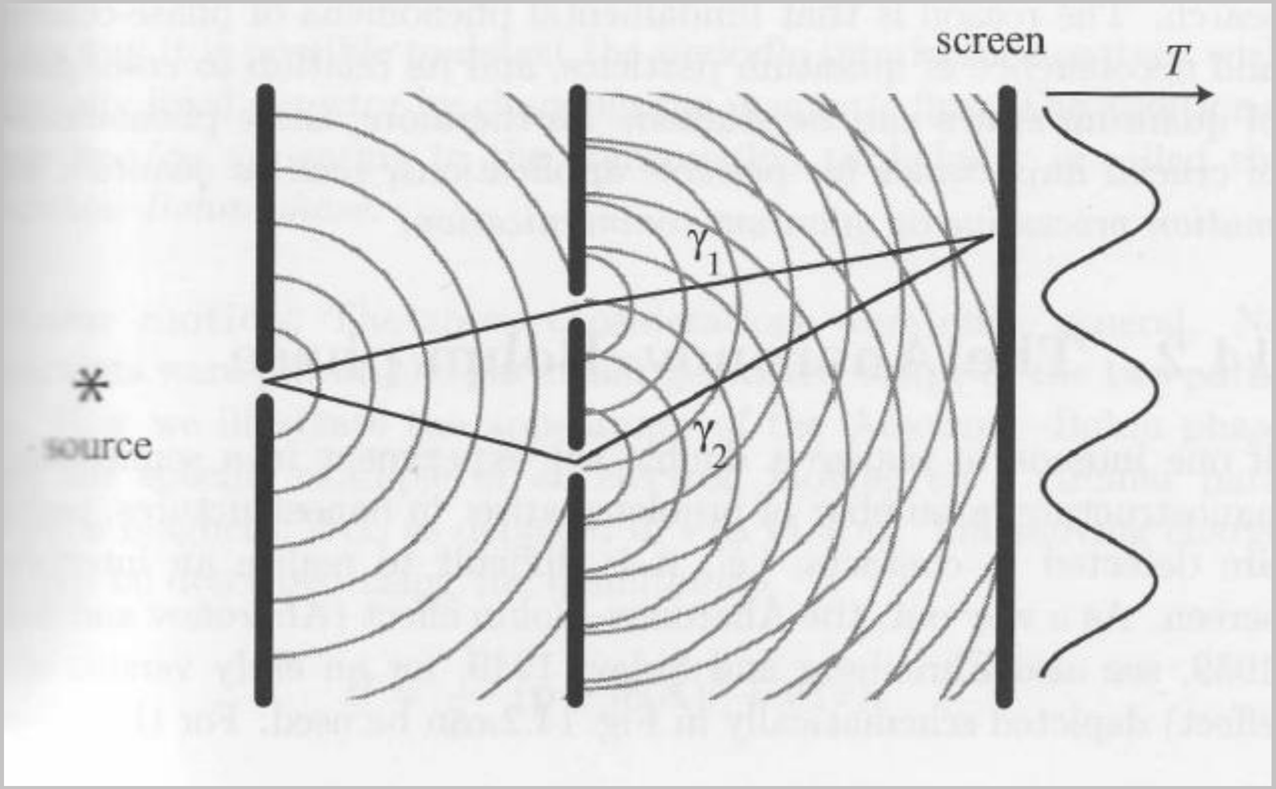
Interferenz von Wellen – Beugung am Doppelspalt



Classical Lagrangian without a magnetic field

In the absence of a magnetic field, the classical equation of movement of a point particle is given by the second Newton's law:

$$m\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}U \quad (\Leftrightarrow m\ddot{x} = -\frac{\partial U}{\partial x} \quad m\ddot{y} = -\frac{\partial U}{\partial y} \quad m\ddot{z} = -\frac{\partial U}{\partial z}) \tag{8.5}$$

The Lagrangian is defined as $L=T-U$, where $T=mv^2/2$ is the kinetic energy and U is the potential energy. We easily verify that the laws of motion written above are identical to ($q_i=x,y,z$):

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0, \tag{8.6}$$

which are the Lagrangian equations of motion. With a magnetic field B the law of motion for a particle with charge q is

$$m\frac{d^2\vec{r}}{dt^2} = q\left(\vec{E} + \frac{d\vec{r}}{dt} \wedge \vec{B}\right). \tag{8.7}$$

It is somewhat tedious but not conceptually difficult to show that the Lagrangian law of motion equation (8.6) is still identical to the equation above if the Lagrangian function is now [COH 77]

$$L = \frac{1}{2}m\left(\frac{d\vec{r}}{dt}\right)^2 + q\frac{d\vec{r}}{dt}\vec{A} - qV(\vec{r}) \tag{8.8}$$

where $\vec{A}$ is the vector potential ($\vec{B} = \vec{\nabla} \times \vec{A}$).

Phase shift due to a magnetic field

With a magnetic field the contribution of the vector potential to the action integrated over a particular path is that of the second term in equation (8.8). From equations (8.4) and (8.8) the time can be eliminated and we immediately obtain

$$\Delta S_A = -e\int_0^t \frac{d\vec{r}}{d\tau}\vec{A}\,d\tau = -e\int_\Gamma \vec{A}\,d\vec{l}\,. \tag{8.9}$$

The contribution of the vector potential to the action along a path Γ thus has a particularly simple form, since it is proportional to the circulation of the vector potential along that path. It is worth noting that in the case of a closed orbit, we can turn this circulation into the magnetic field flux by applying Stoke's theorem, which states that the circulation of a vector around a closed orbit is equal to the flux of the rotational through the corresponding oriented surface. Thus, the probability amplitude phase shift due to a given trajectory around a *closed* orbit is (see equation (8.9))

$$\Delta\varphi = -\frac{e}{\hbar}\oint_\Gamma \vec{A}d\vec{l} = -\frac{e}{\hbar}\oint_S \vec{B}d\vec{S}\,. \tag{8.10}$$

Aharonov-Bohm effect in mesoscopic rings

Although in a mesoscopic structure the magnetic field is applied to the whole substrate containing the mescoscopic samples, so that such devices cannot really be used to demonstrate the action of a vector potential in areas where the magnetic field is absent, a phenomenon like the Aharonov-Bohm effect can be observed. Submit a mesoscopic, coherent semiconductor ring to a magnetic field B applied perpendicularly to the plane of the ring, as illustrated by Figure 8.1.

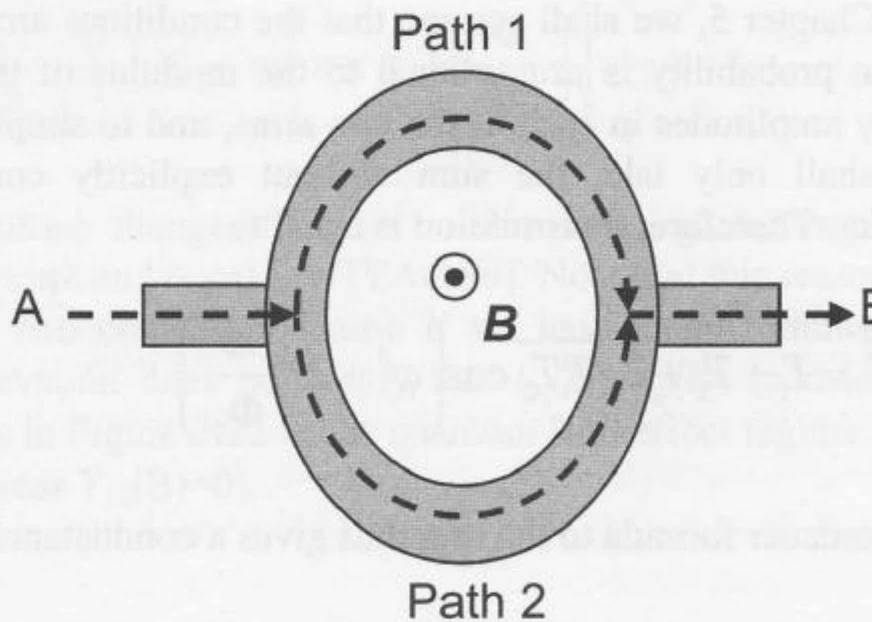


Figure 8.1. *A coherent semiconductor or metal ring submitted to a magnetic field*

Aharonov-Bohm effect in mesoscopic rings

The probability amplitudes passing through both arms of one 1D channel are complex numbers which can be written as

$$t_1 = \sqrt{T_1}\, e^{i\left(\varphi_1^0 + \varphi_1^B\right)} \qquad t_2 = \sqrt{T_2}\, e^{i\left(\varphi_2^0 + \varphi_2^B\right)}, \tag{8.11}$$

where φ_1^0 and φ_2^0 are the phase shifts which would be obtained without a magnetic field (resulting, e.g., from the action of scatterers in the two arms and from wave propagation along paths 1 and B), and T_1 and T_2 are the transmission probabilities from the two arms, respectively. φ_1^B and φ_2^B are the phase shifts induced by the vector potential as expressed by equation (8.9).

According to the results obtained in the previous section, the phase shift difference between the two arms which is due to the action of the vector potential is simply

$$\varphi_1^B - \varphi_2^B = \frac{e}{\hbar}\oint_{ring} \vec{A}\, d\vec{l} = \frac{eBS}{\hbar} = 2\pi\frac{\Phi}{\Phi_0}, \tag{8.12}$$

where Φ_0 is the elementary flux quantum

$$\Phi_0 = \frac{h}{e}. \tag{8.13}$$

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As justified in Chapter 5, we shall assume that the conditions are such that the overall transmission probability is proportional to the modulus of the sum of the complex probability amplitudes in each of the two arms, and to simplify the matter even further we shall only take the sum without explicitly considering any proportionality factor. Therefore, transmission is equal to

$$T = |t_1 + t_2|^2 = T_1 + T_2 + 2\sqrt{T_1 T_2}\cos\left(\varphi^0 + 2\pi\frac{\Phi}{\Phi_0}\right). \tag{8.14}$$

Applying the Landauer formula to the ring thus gives a conductance

$$G = \frac{2e^2}{h}\left(T_0 + T_B \cos\left(\varphi^0 + 2\pi\frac{\Phi}{\Phi_0}\right)\right). \tag{8.15}$$

where $\varphi_0 = \varphi_1^0 - \varphi_2^0$. The oscillatory term is due to the magnetic-field induced interference between the two arms; it can be periodically varied simply by sweeping the magnetic field. Thus, with such a solid-state device we can realize a quantum-mechanical interference experiment "just" by using the device at low temperature and measuring the current as a function of magnetic field. We must obtain oscillations with a universal period, depending only on the elementary flux quantum. Note that we only calculated the contribution arising from probability amplitudes corresponding to a direct transmission, but rigorously we should also take into account the possibility for an electron to travel over one full orbit before being transmitted, or to travel over two orbits, etc. The higher the overall transmission, the larger multiple scattering inside the ring and the more pronounced this effect will be. If the coherence length λ_Φ is large enough, we can thus observe additional oscillations with periodicity Φ_0/N, with N integer, due to paths where the particle travels over more than one circular orbit.

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Yet another surprising point: from equation (8.15) it would seem that the phase φ_0 can take any arbitrary value. However, *this is in fact not the case*. Since we have a two-terminal device in the linear operation regime, it is obvious that if we reverse the sign of the small voltage that we apply to measure the conductance, the current magnitude is conserved even with an applied magnetic field, so that the squared modulus of the transmission probability obeys $|t_{12}(B)|^2=|t_{21}(B)|^2$. However, if we now apply the reciprocity relation we can write for the second term $t_{21}(B)=t_{12}(-B)$, so that $G(B)=G(-B)$. Thus, the conductance of a two-terminal device is an even function of the magnetic field. However, if we take this into account, from equation (8.15) it is easy to find that this implies the striking result $\varphi_0=n\pi$. The phase is "rigid", equal to zero or π, not depending on device geometry! A more correct two-terminal formula is thus

$$G=\frac{2e^2}{h}\left(T_0+T_B\cos\left(n\pi+2\pi\frac{\Phi}{\Phi_0}\right)\right). \qquad (8.16)$$

The only phase changes that can be observed, e.g., by varying the Fermi level position are abrupt and equal to π [YAC 96]. Note that this reasoning does not apply to more than two contacts, because if we have more terminals then the overall current conservation does not imply that $t_{pq}(B)=t_{qp}(B)$ (consider a three-terminal device such as in Figure 3.22 in the quantum Hall effect regime: we obviously have $T_{31}(B)\neq 0$ whereas $T_{13}(B)=0$).

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Aharonov-Bohm effect in the real world

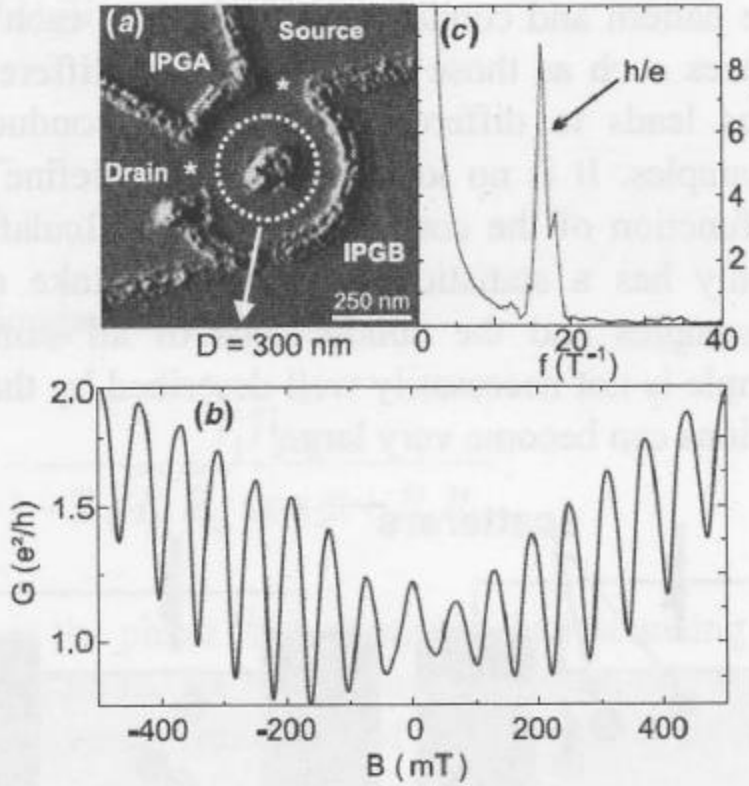

Figure 8.2. *(a)AFM image of an GaAs-GaAlAs heterostructure used to measure (b) Aharanov-Bohm oscillations; the dashed circle in (a) is calculated from the power spectrum (c) (T=25 mK); reprinted with permission from [KEY 02], copyright (2002) by IOP Publishing Ltd*

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Schematischer Aufbau eines Aharonov-Bohm Effekt

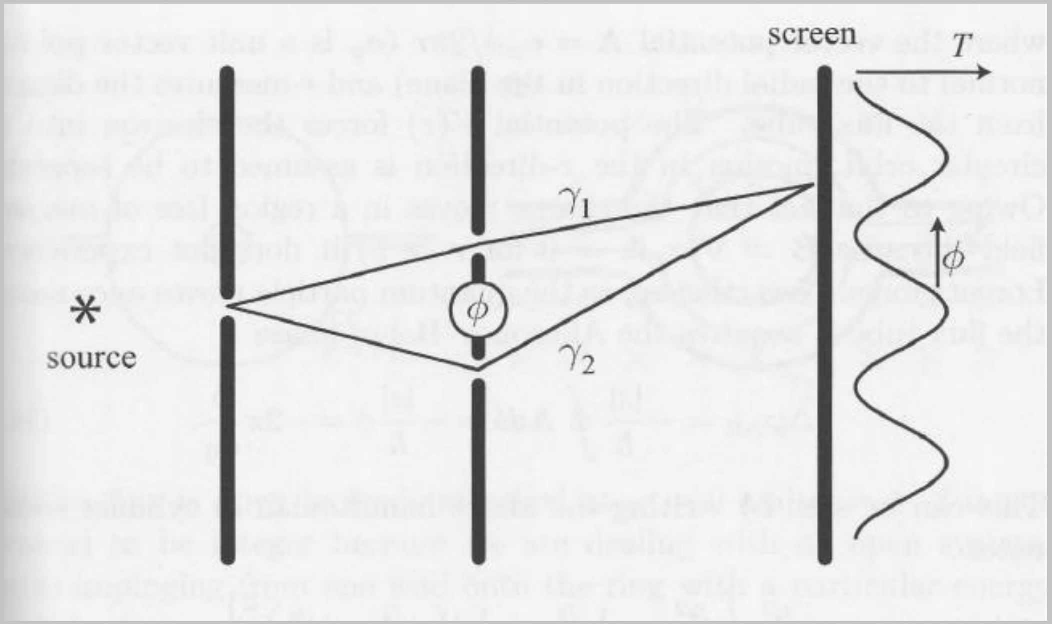


Fig. 14.2 Schematic setup of an Aharonov–Bohm experiment.

$$\theta_i(\phi) = \theta_i(0) - \frac{|e|}{\hbar}\int_{\gamma_i} \mathbf{A}ds.$$

$$\delta(\phi) = \delta(0) - \frac{|e|}{\hbar}\int_{\gamma_1-\gamma_2} \mathbf{A}ds = \delta(0) - 2\pi\frac{\phi}{\phi_0},$$

$$T(\phi) = a_1^2 + a_2^2 + 2a_1a_2\cos\left[\delta(0) - 2\pi\frac{\phi}{\phi_0}\right].$$

$$T(\phi + n\cdot\phi_0) = T(\phi), \text{ with } n \text{ integer.}$$

Aharonov-Bohm und Aharonov-Casher Effekt

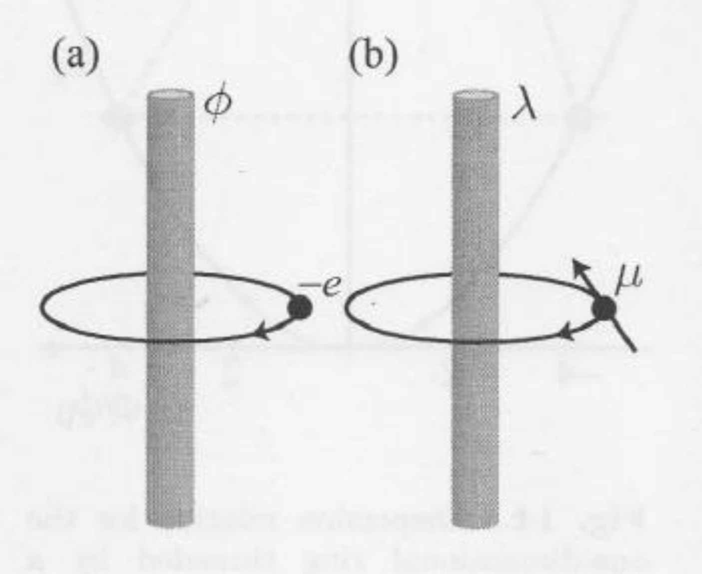
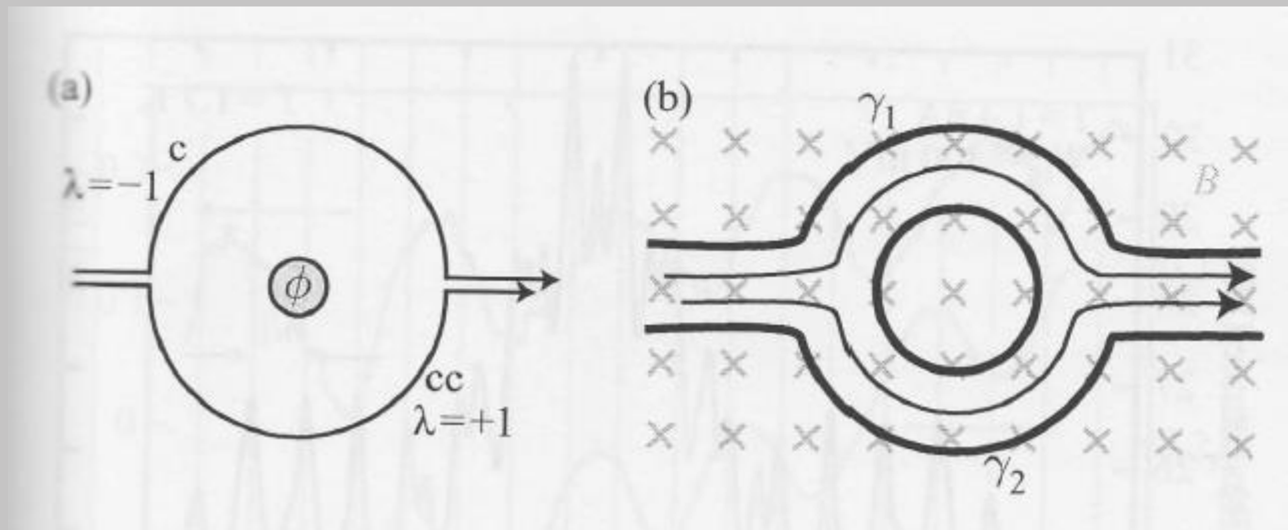


Fig. 14.3 Schematic illustration of the settings for the Aharonov–Bohm effect and its electromagnetic dual, the Aharonov–Casher effect. (a) In case of the Aharonov–Bohm effect a charged particle (e.g., charge $-|e|$) is encircling a magnetic flux tube enclosing the flux ϕ. (b) In case of the Aharonov–Casher effect, an uncharged particle with a magnetic moment (spin) is encircling a tube of constant line charge density λ.

$$H = \frac{1}{2m}(\mathbf{p} + |e|\mathbf{A})^2 + V(r),$$

$$\Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}ds = -\frac{|e|}{\hbar}\phi = -2\pi\frac{\phi}{\phi_0}.$$

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\left(i\frac{\partial}{\partial \varphi} - \frac{\phi}{\phi_0}\right)^2\right] + V(r).$$



$$\ell^{(\lambda)} = \lambda k r_0 - \frac{\phi}{\phi_0}.$$

$$T = |t_1 + t_2|^2 = \frac{1}{2}\left|e^{i(kr_0-\phi/\phi_0)\pi} + e^{i(kr_0+\phi/\phi_0)\pi}\right|^2 = \frac{1}{2}[1+\cos(2\pi\phi/\phi_0)].$$

Aharonov-Bohm Effekt in einem Quantum Ring

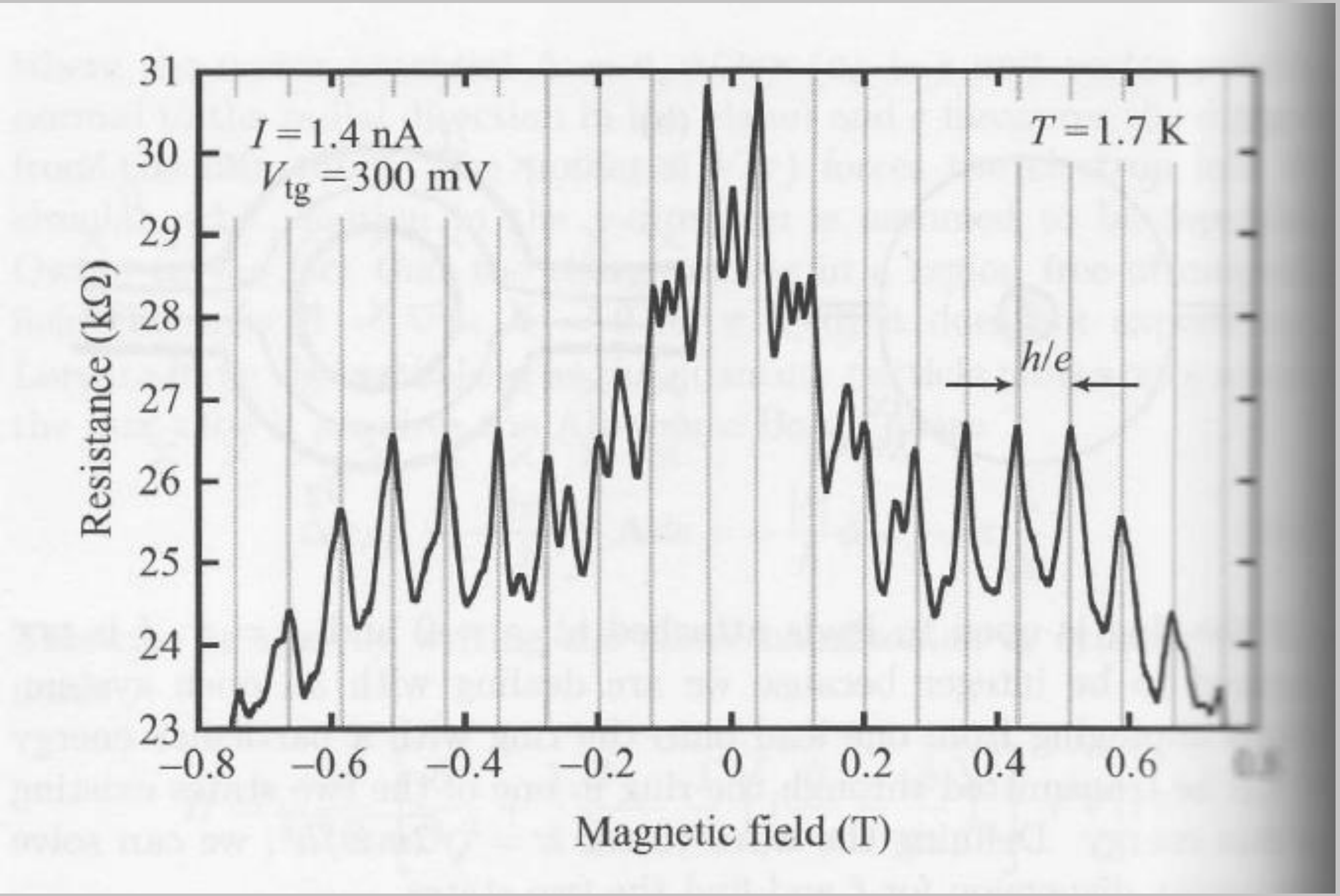


Fig. 14.6 Aharonov–Bohm effect in the quantum ring structure shown in Fig. 6.15(a).

$$\phi = B \cdot A.$$

Altshuler-Aranov-Spivak Oszillationen in einem Metallzylinder

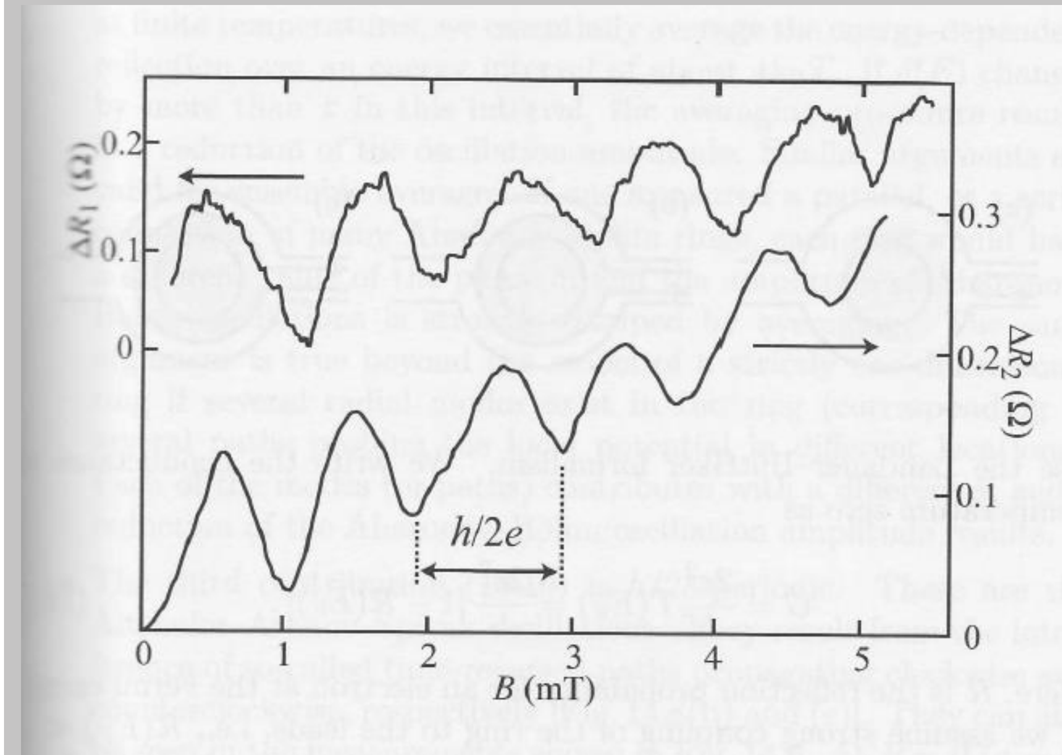


Fig. 14.7 Altshuler–Aronov–Spivak oscillations in a metal cylinder. (Reprinted with permission from Sharvin and Sharvin, 1981. Copyright 1981, American Institute of Physics.)

$$\Delta B = \frac{h/e}{A}.$$

$$\Delta B = \frac{h/(2e)}{A}.$$

Aharonov-Bohm Experiment

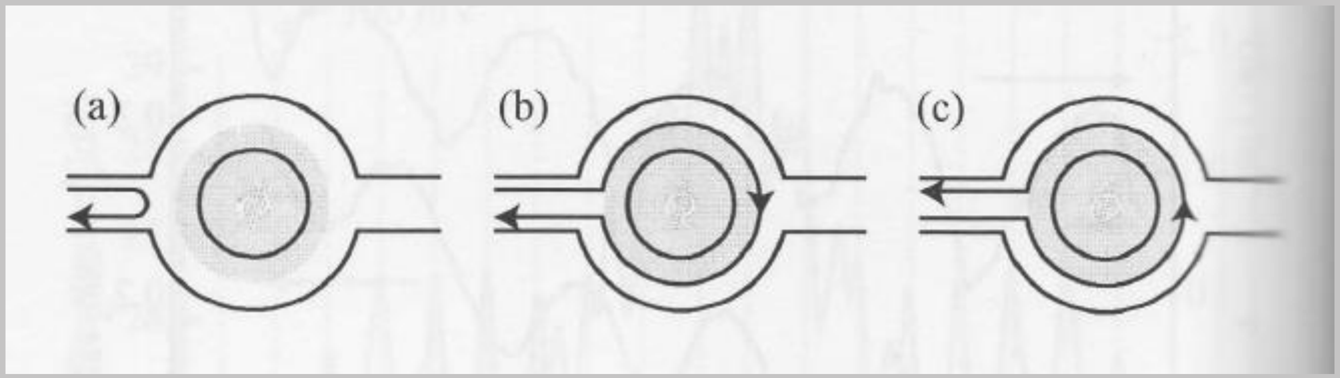


Fig. 14.8 Paths considered for the description of the Aharonov–Bohm experiment in a quantum ring structure. (a) The electron is reflected at the entrance to the ring. (b) The electron is reflected after having explored the ring once in a clockwise direction. (c) The electron is reflected after having explored the ring once in a counter-clockwise direction.

$$G = \frac{2e^2}{h}\mathcal{T}(E_\mathrm{F}) = \frac{2e^2}{h}\left[1 - \mathcal{R}(E_\mathrm{F})\right].$$

$$\begin{aligned}\mathcal{R} &= \left| r_0 + r_1 e^{i\cdot 2\pi\phi/\phi_0} + r_1 e^{-i\cdot 2\pi\phi/\phi_0} + \ldots \right|^2 \\ &= |r_0|^2 + 2|r_1|^2 + \ldots \\ &\quad +4\,|r_0|\,|r_1|\cos\delta\cos\left(2\pi\frac{\phi}{\phi_0}\right) + \ldots \\ &\quad +2\,|r_1|^2\cos\left(4\pi\frac{\phi}{\phi_0}\right) + \ldots\end{aligned}$$

Aharonov-Bohm Oszillationen

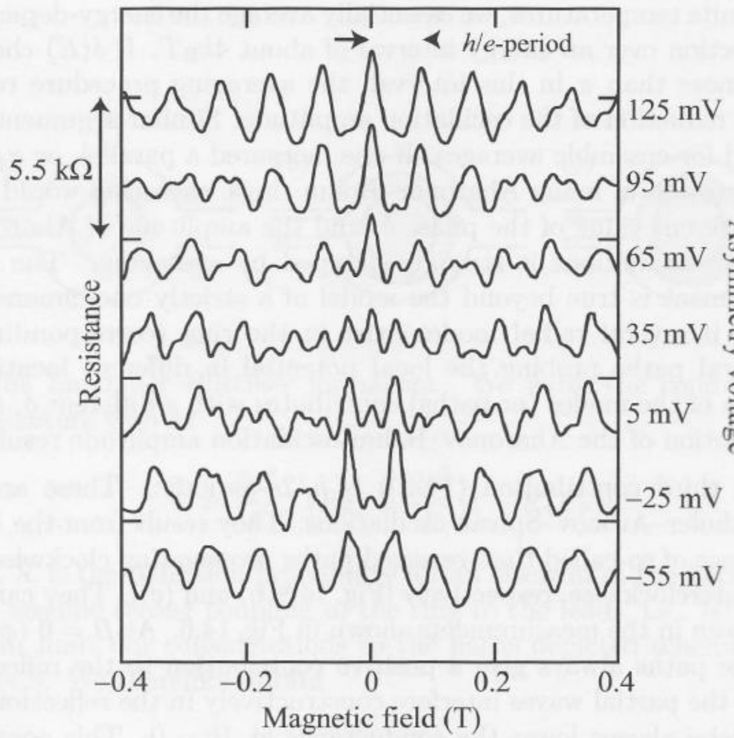


Fig. 14.9 Change of the phase of Aharonov–Bohm oscillations as a function of an asymmetrically applied gate voltage. In the topmost curve a maximum can be seen at $B = 0$, whereas a minimum is observed at $B = 0$ in the bottom curve. At the transition around 35 mV the amplitude of the h/e-periodic oscillations is zero and only the $h/2e$-periodic Altshuler–Aronov–Spivak oscillations can be seen.

$$\mathcal{T} = 1 - \mathcal{R}_{\mathrm{cl}}$$

$$-\sqrt{2(\mathcal{R}_{\mathrm{cl}}^2 - \Delta\mathcal{R}_{\mathrm{cl}}^2)}\cos\delta\cos\left(2\pi\frac{\phi}{\phi_0}\right) - \frac{1}{2}(\mathcal{R}_{\mathrm{cl}} - \Delta\mathcal{R}_{\mathrm{cl}})\cos\left(4\pi\frac{\phi}{\phi_0}\right) -$$

$$\mathcal{T}_{\mathrm{min}} = 1 - \left(\frac{3}{2} + \sqrt{2}\right)\mathcal{R}_{\mathrm{cl}} - \ldots$$

Temperaturabhängiger Magnetowiderstand eines Quantumrings

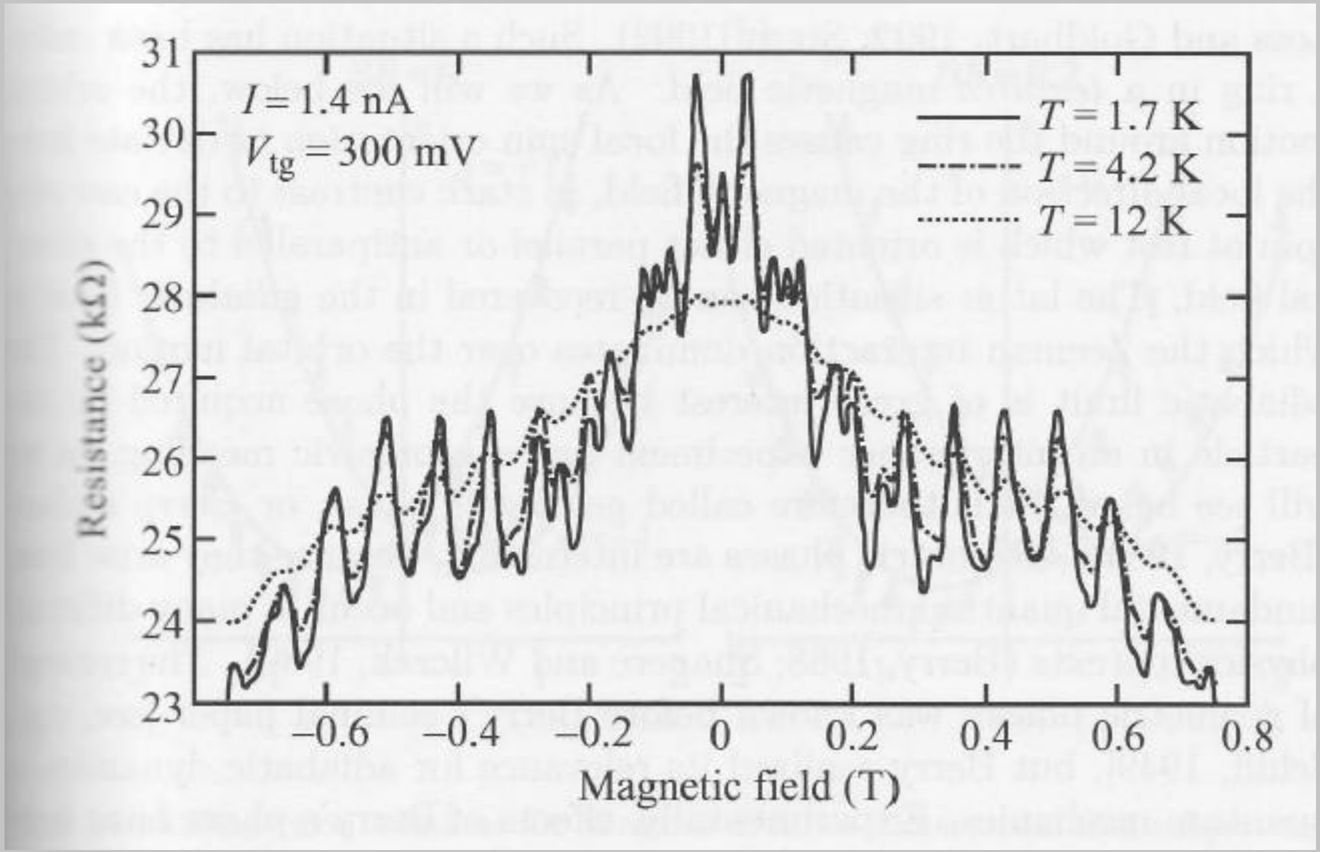


Fig. 14.10 Temperature dependence of the magnetoresistance of a quantum ring.

Temperaturabhängige Amplitude A

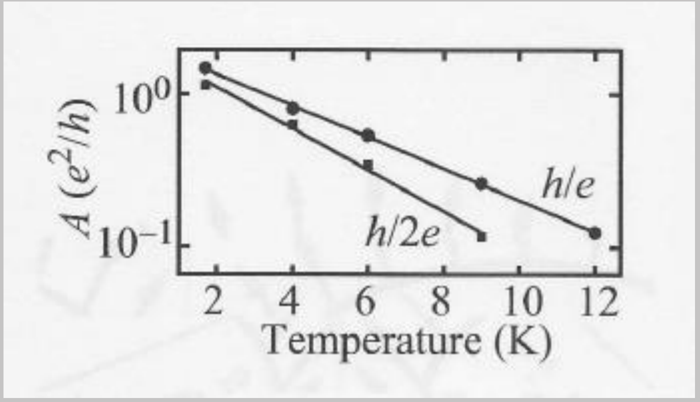


Fig. 14.11 Temperature dependence of the amplitude A of h/e- and $h/2e$-periodic oscillations as determined from a Fourier analysis of the data in Fig. 14.10.

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$$t_1 = \sqrt{T_1}\, e^{i\left(\varphi_1^0 + \varphi_1^B\right)} \qquad t_2 = \sqrt{T_2}\, e^{i\left(\varphi_2^0 + \varphi_2^B\right)}$$

$$\varphi_1^B - \varphi_2^B = \frac{e}{\hbar}\oint_{ring} \vec{A}\,d\vec{l} = \frac{eBS}{\hbar} = 2\pi\frac{\Phi}{\Phi_0}$$

Φ_0 is the elementary flux quantum

$$\Phi_0 = \frac{h}{e}.$$

$$T = \left|t_1 + t_2\right|^2 = T_1 + T_2 + 2\sqrt{T_1 T_2}\cos\left(\varphi^0 + 2\pi\frac{\Phi}{\Phi_0}\right)$$

$$G = \frac{2e^2}{h}\left(T_0 + T_B\cos\left(\varphi^0 + 2\pi\frac{\Phi}{\Phi_0}\right)\right)$$

$$G = \frac{2e^2}{h}\left(T_0 + T_B\cos\left(n\pi + 2\pi\frac{\Phi}{\Phi_0}\right)\right)$$